Martin Knor: Radially maximal graphs

Graph G is *selfcentric* if $diam(G) = rad(G)$, and it is *radially maximal* if $rad(G \cup e) < rad(G)$ for every edge e from the complement of G. My problem is related to radially maximal graphs of radius r with the minimum posssible number of vertices. Well, if we admit selfcentric graphs, then the minimum number is $2r$ and it is attained by the even cycle C_{2r}. For non-selfcentric graphs we have the following conjecture.

Conjecture: Let G be a non-selfcentric radially maximal graph of radius $r \geq 3$ on the minimum possible number of vertices. Then $|V(G)| = 3r - 1$. Moreover $\Delta(G) = 3$, $\delta(G) = 1$ and G is planar.

The conjecture was verified for $r = 3$, and it was proved that there are only 2 graphs of this form. For $r > 3$ nothing is known.

To show you some examples of these graphs, consider cartesian product $P_{2r-1} \times P_2$, and contract r-1 copies of P_2 (each copy into one point) at one end of the ledder. Another example is C_8 with vertices $v_0, v_1, \ldots v_7$ with 6 extra vertices as follows. Two paths of length 2 are glued, by their endpoints, one to v_0 and the other to v_4; and two paths of length 1 are glued by their endpoints one to v_2 and the other to v_6.