Michael Kubesa: Factorization of K_{2n} into "brooms" B_m

Find all pairs n, m such that there exists a factorization of K_{2n} into "brooms" B_m (we get broom B_m by adding m leaves into one endvertex of a path of length $2n - m - 1$). It is known that such a factorization exists if $n = 2k + 1$ and $1 \leq m < k + 1$. Other cases are unsolved.