Martin Mačaj: Minimum $k$-GC graphs.

A connected graph $G$ is $k$-geodetically connected ($k$-GC) if the removal of at least $k$ vertices is required to increase the distance between at least one pair of vertices or reduce $G$ to a single vertex.

It is known that if $n > 2k$, then any minimum (i.e., with the least possible number of edges) $k$-GC graph of order $n$ has at most $nk - k^2$ edges. A conjecture of Ján Plesník says that there exists a real constant $c$ such that if $n > ck$, then any minimum $k$-GC graph of order $n$ has exactly $nk - k^2$ edges.

In order of simplicity we will say that $k$-GC graph of order $n > 2k$ and size $m < nk - k^2$ is small:

- find a non-bipartite small graph,
- find a small graph with $\Delta(G) \geq 2k$,
- find a small graph with $\text{diam}(G) > 3$,
- prove Plesník’s conjecture ($c$ has to be at least $3 + \sqrt{5}$).