Martin Mačaj: Minimum k-GC graphs.

A connected graph G is k-geodetically connected (k-GC) if the removal of at least k vertices is required to increase the distance between at least one pair of vertices or reduce G to a single vertex.

It is known that if $n > 2k$, then any minimum (i.e., with the least possible number of edges) k-GC graph of order n has at most $nk-k^2$ edges. A conjecture of Ján Plesník says that there exists a real constant c such that if $n > ck$, then any minimum k-GC graph of order n has exactly $nk - k^2$ edges.

In order of simplicity we will say that k-GC graph of order $n > 2k$ and size $m < nk - k^2$ is *small*:

- find a non-bipartite small graph,
- find a small graph with $\Delta(G) \geq 2k$,
- find a small graph with diam $(G) > 3$,
- prove Plesník's conjecture (c has to be at least $3 + \sqrt{5}$).

For more details see J. Plesník: Towards minimum k-geodetically connected graphs. Networks Vol. 41(2), 73-82 2003.