Ring's maps $M_n(p, q)$

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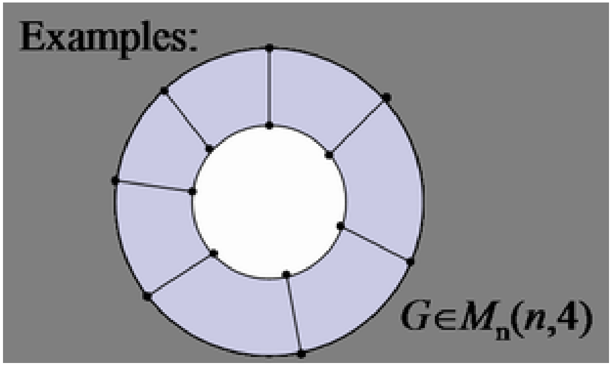
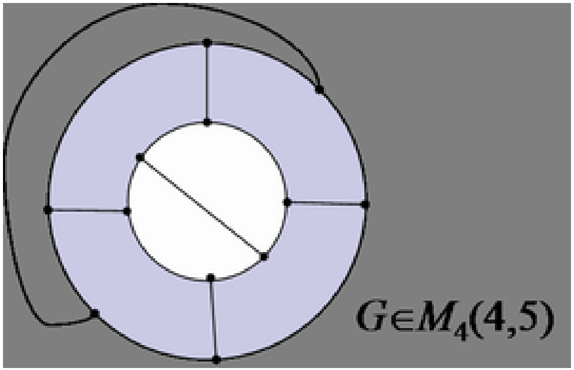
Definition 1 *$M_n(p,q)$ - set of all 3-regular plane graphs having only p-gonal and q-gonal faces such that q-gonal faces form a ring R_n of n q-gons. ($p \geq 3$, $q \geq 4$)*
(Deza, Grishukhin 02)





Problem

To find all values n with $M_n(7,5) \neq \emptyset$.

For finding this maps we used duals of this maps. Duals of this maps are graphs with vertices of degree 7 and 5.

This maps have in chemistry special name - azzulenoids.

Theorem 1 *The set $M_n(7,5)$ is nonempty for following particular values:*

a) $n \in \{30, 36, 42\}$

b) $n = 28 + 12k, k > 0$

c) $n = 32 + 8k, k > 0$

d) $n = 32 + 20k, k > 0$

e) $n = 28 + 28k, k > 0$

The set $M_n(7,5)$ is empty for $n \in \{29, 31, 33, 34, 35, 37\}$

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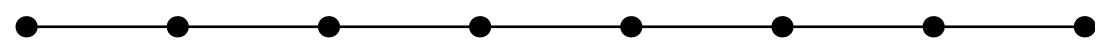
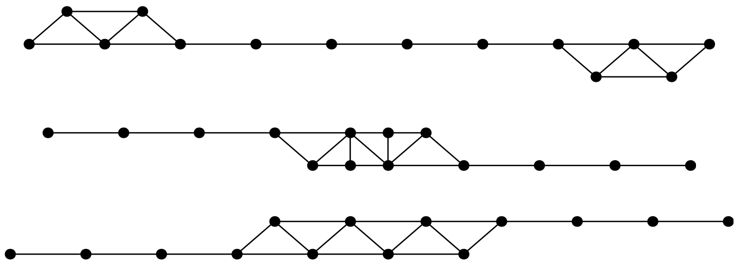
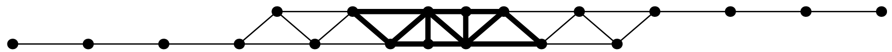
I_{28}



I_{40}



I_{40+12k}



Some relations

$\lceil \frac{n-4}{3} \rceil \leq k_i \leq \lfloor \frac{n-12}{2} \rfloor$ — k_i - *number of vertices of degree 7 in I*

$\lceil \frac{n-12}{2} \rceil \leq k_o \leq \lfloor \frac{2n-32}{3} \rfloor$ — k_o - *number of vertices of degree 7 in O*

$k_i + k_o = n - 12$

$c_i = 3k_i - n + 4$ — c_i - *number of faces incident only with inner vertices*

$c_o = 3k_o - n + 4$ — c_o - *number of faces incident only with outer vertices*

$c_i + c_o = n - 28$

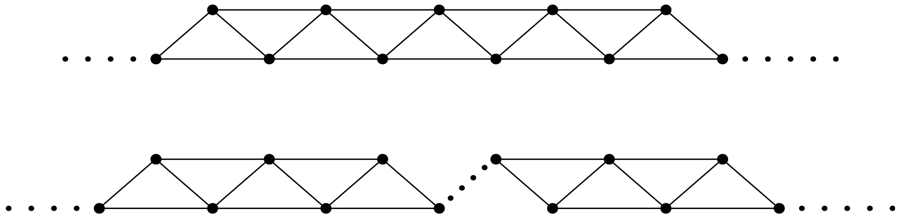
n	k_i	k_o	c_i	c_o
29	8	9	-1	2
31	9	10	0	3
33	10	11	1	4
34	10	12	0	6
34	11	11	3	3
35	11	12	2	5
37	11	14	0	9
37	12	13	3	6
⋮	⋮	⋮	⋮	⋮

n = 37

a) $k_i = 11, k_o = 14, c_i = 0, c_o = 9$

I_{37}

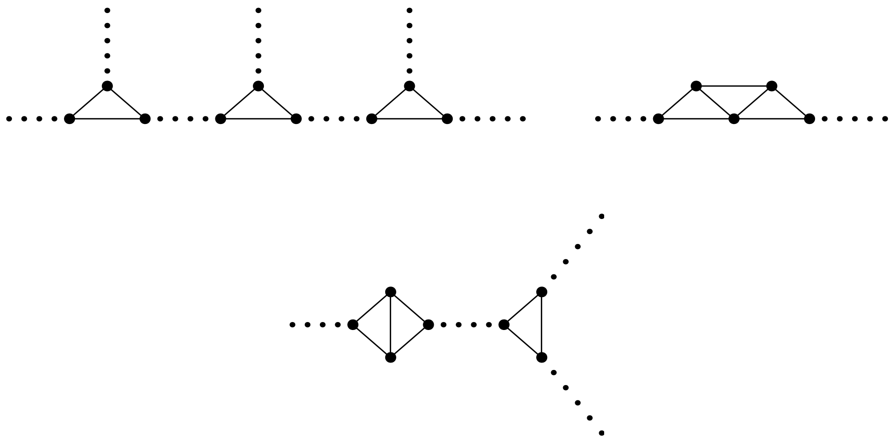
O_{37}



$\mathbf{n = 37}$

b) $k_i = 12, k_o = 13, c_i = 3, c_o = 6$

I_{37}

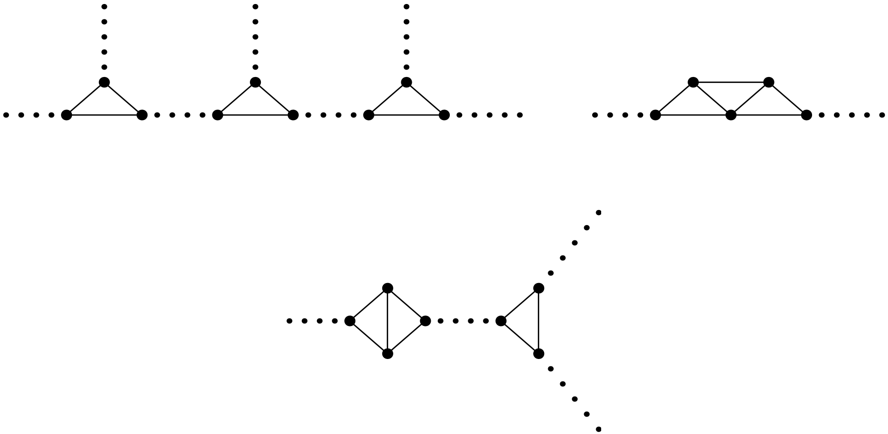


$\mathbf{n = 34}$

$a)\ k_i = 10, k_o = 12, c_i = 0, c_o = 6$

$b)\ k_i = 11, k_o = 11, c_i = 3, c_o = 3$

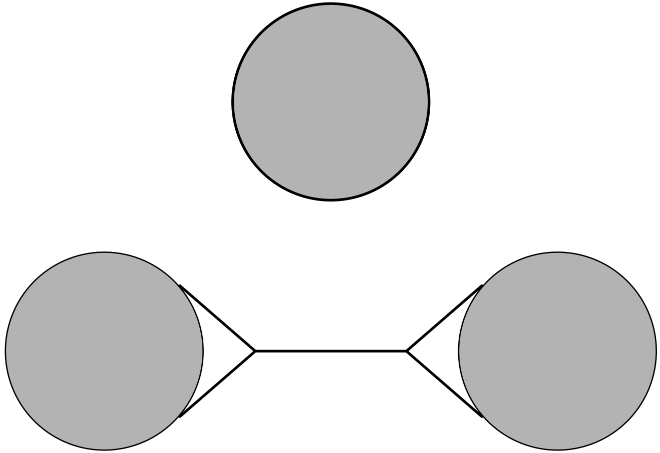
I_{34}, O_{34}



$\mathbf{c_i = 0, c_o = n - 28}$

I_n

O_n



$n = 46+3v_7$

$n = 40+3v_7$

- *open for some* $n \equiv 0 \pmod 4$
- *open for all* $n \equiv 2 \pmod 4$ *(except 30, 34 and 42)*
- *open for all* $n \equiv 1 \pmod 2$ *(except 29, 31, 33, 35 and 37)*
- *we don't determine* $|M_n(7,5)|$
- *in all positive cases we have* I *and* O *isomorphics*

Thanks for your attention