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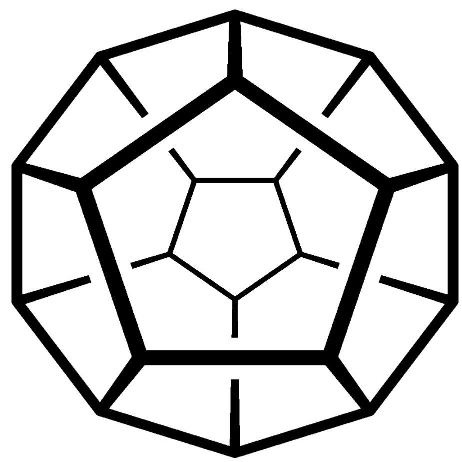
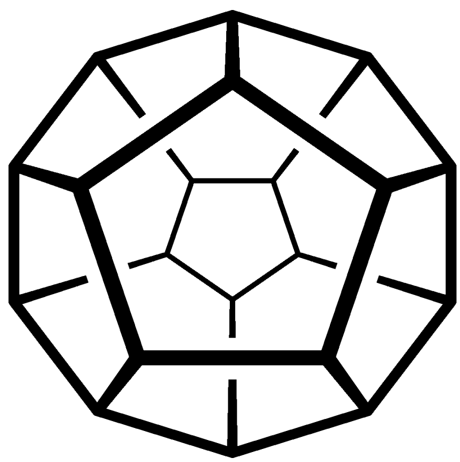


GRAPHS 2005

40th Czech and Slovak Conference on Graph Theory and Combinatorics

May 30 – June 3, Budmerice, Slovakia

Abstracts



GRAPHS 2005

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Abstracts

http://www.math.sk/csgt2005

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Contents

History

The following list contains all 39 previously held Czechoslovak (since 1993 called Czech and Slovak) conferences on graph theory and combinatorics.

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1977 Jenišov
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1979 Nová Ves u Branžeže
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2001 Sedmihorky
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2003 Javorná
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Invited talks

On labelings of graphs

Martin Bača

A *labeling* of a graph is any map that carries some set of graph elements to numbers (usually to the positive integers). Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of "sum" property. An *edge-magic total labeling* on a graph with v vertices and e edges is defined as a one-to-one map taking the vertices and edges onto the integers $1, 2, \ldots, v + e$ with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge. If the sums of the labels on the edges and the labels of their endpoints form an arithmetic progression starting from a and having common difference d then the labeling is called (a, d)-*edge-antimagic total.*

We will present edge-magic and edge-antimagic labelings for some families of graphs.

Representing finite groups via regular actions on combinatorial structures

Robert Jajcay

Every finite group has a regular representation on its own elements via left or right multiplication. The problem we shall discuss in our lecture is the one of imposing a combinatorial structure on the set of elements of the group in such a way that the full automorphism group of the resulting structure is exactly equal to the original regular representation of the group. Various combinatorial structures will be discussed - graphs, digraphs, maps, incidence structures, or hypergraphs. In each case, the main goal is the classification of all those finite groups that admit a representation via a structure from the given class.

Matroid intersection and 2-walks in tough graphs

Tomáš Kaiser

A deep topological extension of the Matroid intersection theorem, proved recently by Aharoni and Berger, deals with the intersection of a matroid and a simplicial complex. We discuss applications of this theorem in graph theory. The necessary background in topological combinatorics and matroid theory will be included. We also show how to use the result to derive a sufficient condition for a graph to have a spanning tree satisfying given local constraints. Finally, we point out a connection to the problem of the existence of spanning 2-walks (i.e., closed walks visiting each vertex once or twice) in sufficiently tough graphs.

Applications of superposition in graph theory

Martin Kochol

We give survey of several results about nowhere-zero flows in graphs, graph coloring, snarks, complexity of edge coloring and flows, dominating and hamiltonian circuits on graphs. The common feature of these results is a constructive technique in graph theory, called superposition. We also outline some open problems in the area.

Geometric representations of graphs

Jan Kratochvíl

Geometric representations of graphs are intensively studied both for the practical motivation and interesting theoretical properties. I will survey recent results on intersection and contact graphs of geometrical objects in the plane.

Regular embeddings of complete bipartite graphs

Roman Nedela

(joint work with S.F. Du, G. Jones and M. Škoviera)

A 2-cell embedding of a graph into a compact connected surface is called regular, if the respective group of automorphisms of the embedding acts transitively (that means regularly) on the set of arcs of the graph. Several authors considered the problem of classification of regular embeddings of complete bipartite graphs. However, the classification is, in general, not known. In our talk we survey known results and show connections between regular embeddings of $K_{n,n}$ and other structures such as products of cyclic groups, skew-morphisms of cyclic groups, Schur rings e.t.c.

Reaction graphs of combinatorial configurations

Alexander Rosa

Motivation for studying reaction graphs came originally from chemistry. A reaction graph of a combinatorial configuration C has as its vertices all nonequivalent realizations of C. Its edges correspond to "small" well-defined changes effecting a transition from one realization of C to another. Typically, these graphs have a large number of vertices and a large degree of symmetry. Interesting questions that one can ask about these graphs include connectivity, Hamiltonicity and automorphisms. We illustrate the concepts and problems on examples of reaction graphs of the Fano plane and of the $(K_4 - e)$-design of order six.

Coloring squares of planar/sparse graphs

Riste Škrekovski

(joint work with Z. Dvořák, D. Kráľ, P. Nejedlý and M. Tancer)

The *square* G^2 of a graph G is the graph with the same vertex set in which two vertices are joined by an edge if their distance in G is at most two. The problem of colouring the square of a graph naturally arises in connection with the problem of assigning frequencies to transmitters and the distance labelings, which have been studied intensively.

We consider two special cases of the problem, first when the graph is planar and with large girth, and second when it has maximum degree 3 but its maximum average degree is small. In the talk will be presented some new results and problems.

Contributed talks

Hamiltonian Cayley maps of K_n

Marcel Abas

A map is called Hamiltonian if the boundary of every face of the map is a Hamiltonian circle. In this talk we show that for each even n there exists a Hamiltonian map M with underlying graph K_n such that M is a Cayley map.

Reduction theorem for bicolored prime graphs

Roman Čada

(joint work with Francois Genest and Tomáš Kaiser)

We present an extension of the reduction theorem for prime graphs due to A. Bouchet for bicolored graphs and discuss some related topics.

Applications of graph similarity measures for generalized trees

Matthias Dehmer

(joint work with Frank Emmert Streib and Jürgen Kilian)

Concerning the tremendous amount of information available online, developing methods for mining the graph structure of web-based documents is an important research field. In this talk we will focus on graph theoretic methods for exploring the graph-based structure of web-based documents. First, we present a novel approach for measuring the structural similarity of generalized trees.

Second, we apply agglomerative clustering methods to the obtained similarity matrices. This leads to important applications in Web Structure Mining and other graph similarity problems dealing with generalized trees.

Oberwolfach rectangular table negotiation problem

Dalibor Fronček

There are many modifications of the well known Oberwolfach problem. One of them is the bipartite version which asks: For what values k, n it is possible to decompose the complete bipartite graph $K_{2n,2n}$ into graphs isomorphic to mC_{2k}, where $m = 2n/k$? This problem was solved by W.-L. Piotrowski, and can be also described as follows. Suppose we have two delegations with n people each, and we want to find a seating arrangement over n nights such that every night the members of the delegations sit alternately around m round tables, each table accommodating $2k$ people, and every person sits next to each member of the other delegation exactly once. One could agree that while such an arrangement is good for social occasions, it is not particularly suitable if we assume that the delegations are involved in some negotiations. Then it would be more natural to have rectangular tables with members of each delegation sitting along one of the long sides of the table while the tables would not be too big. Even if we have tables with three people on each of the two long sides of the table, it is reasonable to assume that the people sitting at the opposite corners cannot easily communicate with each other. Therefore, we may translate this modification into terms of graph decompositions as follows.

We say that a graph B has a *G-decomposition* if there are subgraphs $G_0, G_1, G_2, \dots, G_s$ of B, all isomorphic to G, such that each edge of B belongs to exactly one G_i. If the graph G (more precisely, each $G_i, i = 0, 1, \dots, s-1$) contains all vertices of B, then we say that B has a *G-factorization*.

Let $H(k, 3)$ be a bipartite graph with bipartition $X = x_1, x_2, \dots, x_k, Y = y_1, y_2, \dots, y_k$ and edges $x_1y_1, x_1y_2, x_ky_{k-1}, x_ky_k$, and $x_iy_{i-1}, x_iy_i, x_iy_{i+1}$ for $i = 2, 3, \dots, k-1$. We always assume that $k \geq 3$. We want to characterize all complete bipartite graphs $K_{n,n}$ that can be factorized into factors isomorphic to $G = mH(k, 3)$, where $mH(k, 3)$ is the graph consisting of m disjoint copies of $H(k, 3)$. Since the number of edges of G equals $m(3k-2)$ and the number of its vertices in each partite set equals mk, the necessary conditions are $n = mk$ and $n^2 \equiv 0 \pmod{m(3k-2)}$. We will show that then m must be a multiple of $3k-2$ which yields $n \equiv 0 \pmod{k(3k-2)}$. We will show that these necessary conditions are also sufficient for the existence of a G-factorization of $K_{n,n}$.

Rings maps $M_n(7,5)$

Róbert Hajduk

(joint work with Roman Soták)

We study existence of maps $M_n(p,q)$ (maps with ring of n $q-gons$ whose inner and outer domain are filled by p-gons). In succesion on [1] we study existence of maps $M_n(7,5)$. We find some next values of n for which no maps $M_n(7,5)$ exists. Namely for $n = 29, 31, 33, 35, 37$. We are focused on maps $M_n(7,5)$ with inner domain filled by the chain of pentagons too.

[1] M. Deza, V. P. Grishukhin: *Maps of p-gons with a ring of q-gons*, Bull. Inst. Comc. Appl. **34** (2002) 99-110.

A note on families of integral trees of diameter $4, 6, 8$ and 10

Pavel Híc

(joint work with Milan Pokorný)

A graph G is called integral if all the zeros of the characteristic polynomial $P(G;x)$ are integers. The notion of integral graphs was introduces by F. Harary and A.J.Schwenk in 1974. There are many results on integral trees of diameter less or equal 10. In this paper some new families of integral trees of diameter 6 and 8 are given. All of these classes are infinite and they are different from the classes that were known before.

AMS Subject Classification (2000): 05C50, Keywords: Integral Tree, Characteristic Polynomial.
Supported by 1/10001/04 VEGA grant.

Edge closure concept in claw-free graphs

Přemysl Holub

(joint work with Jan Brousek)

Ryjáček introduced a claw-free closure concept based on local completion of a locally connected vertex of a claw-free graph. Several results on the stability of forbidden subgraphs were shown. In this paper, we give a variation of a cycle closure concept introduced by Broersma and Ryjáček for C_4. This closure is based on a local completion of a locally connected edge of a claw-free graph. The closure is uniquely determined and preserves the value of the circumference of a graph. Moreover the stability of forbidden subgraphs is proved.

On-line arbitrarily vertex decomposable trees

Mirko Horňák

(joint work with Zsolt Tuza and Mariusz Woźniak)

A tree T is *arbitrarily vertex decomposable* (avd for short) if for any sequence $(t_1, \ldots, t_k)$ of positive integers adding up to $|V(T)|$ there is a sequence $(T_1, \ldots, T_k)$ of vertex-disjoint subtrees of T such that $|V(T_i)| = t_i$ for $i = 1, \ldots, k$. Though it is known that $\Delta(T) \leq 4$ for any avd tree T, the problem of recognising avd trees seems to be hard. In an on-line version of the problem members of (a random) sequence $(t_1, \ldots, t_k)$ are coming one by one and the position of a tree T_i has to be chosen in the moment when t_i arrives without a possibility of changing it in the future. A complete characterisation of on-line avd trees is given.

Minimal eccentric sequences with least eccentricity four

Pavel Hrnčiar

(joint work with Gabriela Monoszová)

A sequence of positive integers is called *eccentric* if there is a graph which realizes considered sequence as the sequence of the eccentricities of its vertices. An eccentric sequence is called *minimal* if it has no proper eccentric subsequence with the same number of distinct eccentricities.

All minimal eccentric sequences of type $(4^{\alpha}, 5^{\beta})$ are known. We present, among others, all minimal eccentric sequences of type $(4^{\alpha}, 5^{\beta}, 6^{\gamma})$ for $\alpha \geq 3$.

Graphs in combinatorial theory of semigroups

Tatiana Jajcayová

We introduce an essential link between the theory of inverse semigroups presentations (and related problems) and the graph concepts. We characterize Schutzenberger graphs (Cayley graphs of R-classes) of an important and useful class of inverse semigroups - HNN extensions, and use this characterization to answer some structural and algorithmic questions.

H-hamiltonicity of 3-valent polyhedral graphs

Ján Karabáš

(joint work with Roman Nedela)

Mednykh and Vesnin in a serie of papers introduced a construction of hypereliptic 3-manifolds from hamiltonian 3-valent polyhedral graphs. Later these results were generalised to construct other types of 3-manifolds from 3-valent polyhedral graphs containing spanning subgraphs homeomorphic to Θ-graph, and K_4. It follows that similar constructions can be obtained if we replace Θ-graph or K_4 by a general 2-connected spanning graph. The above results led us to the following problem.

Let H be either a loop or a cubic 2-connected planar multigraph H, denote the set of these graphs by $\mathcal{H}$. We say that a 3-valent polyhedral graph G is *H-hamiltonian* if G contains a spanning subgraph homeomorphic to H. The vertices of degree 3 in the spanning subgraph (homeomorphic to H) will be called *essential*. The minimum number of essential vertices taken through all k-connected spanning subgraphs, $k \in \{2, 3\}$, will be called a *hamiltonian deficiency* of G and will be denoted by $\delta_k(G)$. We show that both δ_2 and δ_3 are unbounded.

Let $H, K \in \mathcal{H}$. We say that $H \leq K$ if G is H-hamiltonian implies that G is K-hamiltonian for every 3-valent polyhedral graph G. We shall study some properties of the poset $(\mathcal{H}, \leq)$. A sub-poset induced by the set of 3-valent polyhedral graph can be considered.

Partition problems and kernels of graphs

Peter Katrenič

Let $\tau(G)$ denote the number of vertices in a longest path of a graph $G = (V, E)$. A subset K of V is called a *P_n-kernel* of G if $\tau(G[K]) \leq n - 1$ and every vertex $v \in V(G - K)$ is adjacent to an end-vertex of a path of order $n - 1$ in $G[K]$. A partition A, B of V is called an (a, b) partition if $\tau(G[A]) \leq a$ and $\tau(G[B]) \leq b$. We show that that every graph has a P_9-kernel and for every $n \geq 364$ there exists a graph G that does not contain any P_n-kernel.

Single source multiroute flows and cuts on uniform capacity networks

Petr Kolman

(joint work with Henning Bruhn)

An instance of the *single source flow problem* for a graph $G = (V, E)$ consists of a source vertex $s \in V$ and k sinks $t_1, \dots, t_k \in V$; we denote it $\mathcal{I} = (s; t_1, \dots, t_k)$. In the single source multicommodity *multiroute* flow problem, we are given an instance $\mathcal{I} = (s; t_1, \dots, t_k)$ and the objective is to maximize the total amount of flow that is transferred from the source to the sinks so that the capacity constraints are obeyed and, moreover, the flow of each commodity is an h-route flow (an h-route flow is a non-negative linear combination of elementary h-flows where an elementary h-flow is a flow along h edge disjoint paths between the source and the sink, each path carrying a unit of flow). An *h-disconnecting cut* for $\mathcal{I}$ is a set of edges $F \subseteq E$ such that no $s - t_i$ pair is h-connected in $(V, E - F)$.

We establish a max-flow min-cut theorem for the single source multiroute flow and the minimum disconnecting cut on networks with uniform capacities. In particular, we show that the max-flow is within $2h - 2$ of the min-cut, independently of the number of commodities; we also describe a $2(h - 1)$-approximation algorithm for the minimum h-disconnecting cut problem. The theorem follows from another result that is of its own interest. Given an instance $\mathcal{I} = (s; t_1, \dots, t_k)$ such that each $s - t_i$ pair is h-connected, the maximum classical flow between s and t_i's is at most $2(1 - 1/h)$-times larger than the maximum multiroute flow between s and t_i's and this is the best possible bound. This is in contrast with the situation of general multicommodity multiroute flows where the ratio depends linearly on the number of commodities even for $h = 2$.

Magic labelings of regular graphs

Petr Kovář

A vertex magic total (VMT) labeling of a graph $G(V, E)$ is defined as one-to-one mapping from $V \cup E$ to the set of integers $1, 2, ..., |V| + |E|$ with the property that the weights (sums of the label of a vertex and the labels of all edges incident to this vertex) are equal to the same constant for all vertices of the graph. An (s, d)-vertex antimagic total (VAMT) labeling of a graph $G(V, E)$ is defined as one-to-one mapping from $V \cup E$ to the set of integers $1, 2, ..., |V| + |E|$ with the property that the weights form an arithmetic progression starting at s with difference d.

J. MacDougall conjectured that any regular graph with the exception of K_2 and $2K_3$ has a VMT labeling. In the talk we present recent results on VMT and VAMT labelings of certain even-regular graphs and on VMT labelings of certain odd-regular graphs.

Keywords: vertex magic total labeling, vertex antimagic total labeling, Kotzig arrays

Factorizations of the complete graph K_{2n} into isomorphic spanning trees with given diameters

Tereza Kovářová

We introduce the following results on spanning tree factorizations of the complete graph K_{2n}. For any d, $3 \leq d \leq 2n - 1$, there exists a factorization of K_{2n} into isomorphic copies of a tree with diameter d. Further we give the completed classification of caterpillars with diameter four with respect to an isomorphic factorization of K_{2n}. As a tool for factorizations are used various graceful-type labelings, namely symetric graceful labelings, blended labelings, fixing labelings and swapping labelings.

Channel assignment problem with variable weights

Daniel Kráľ

Distance constrained labelings of graphs form an important model for radio frequency assignment problems. An $L(p_1, ..., p_k)$-labeling of a graph G for integers $p_1, \ldots, p_k$ is a labeling of its vertices by non-negative integers such that the labels of two vertices at distance i differ by at least p_i. The least number K for which there is a proper $L(p_1, ..., p_k)$-labeling by integers between 0 and K is denoted denoted $\lambda_{p_1,...,p_k}(G)$. Note that $\lambda_{1,...,1}(G) + 1$ is the chromatic number of the k-th power of G.

Griggs and Jin studied the dependency of $\lambda_{p_1,...,p_k}(G)$ on the parameters $p_1, \ldots, p_k$ (when G is a fixed finite or infinite graph). We address this problem in a more general setting using a notion of lambda-graphs and prove several conjectures posed by Griggs and Jin: Piecewise Linearity Conjecture, Coefficient Bound Conjecture and Delta Bound Conjecture.

Edge-coloring of multigraphs

Naďa Krivoňáková

(joint work with Martin Kochol and Silvia Smejová)

We introduce a monotone invariant $\pi(G)$ on graphs and show that it is an upper bound of the chromatic index of graphs. Moreover, there exist polynomial time algorithms for computing $\pi(G)$ and for coloring edges of a multigraph G by $\pi(G)$ colors. This generalizes the classical edge-coloring theorems of Shannon and Vizing.

Keywords: Edge-coloring, Supermultiplicity of graphs, r-ordering of graphs

Factorizations of complete graphs into caterpillars of diameter 5

Michael Kubesa

(joint work with Dalibor Fronček, Tereza Kovářová and Petr Kovář)

We present a new inductive method for factorizations of K_{2n} into spanning trees that is based on previously known labelings, namely blended, fixing or swapping labelings. Using the method we completed the characterization of caterpillars with diameter 5 that factorize K_{2n}.

Paths decomposition of complete multidigraph

Mariusz Meszka

(joint work with Zdzisław Skupień)

The general conjecture says that the complete n-vertex multidigraph ${}^{\lambda}\mathcal{D}K_n$ (ie. the multidigraph obtained by replacing each arc of the complete digraph $\mathcal{D}K_n$ of order n by λ arcs) is decomposable into directed paths of arbitrarily prescribed lengths provided that the lengths sum up to the size $\lambda\, n\,(n-1)$ of ${}^{\lambda}\mathcal{D}K_n$, unless all paths are hamiltonian and either $n=3$ and λ is odd or $n=5$ and $\lambda=1$.

Supporting results for the conjecture will be presented, especially in the case when all required paths are to be nonhamiltonian.

Reducible graph properties

Peter Mihók

A graph property is any isomorphism closed class of simple graphs. A graph property is induced-hereditary, if it is closed under taking induced-subgraphs and additive if it is closed under disjoint unions. The set M^a of all additive induced-hereditary properties partially ordered by set-inclusion forms a distributive lattice. Let $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$ be any properties of graphs, a *vertex* $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$*-partition* of a graph G is a partition $(V_1, V_2, \ldots, V_n)$ of $V(G)$ such that for each $i = 1, 2, \ldots, n$ the induced subgraph $G[V_i]$ has the property $\mathcal{P}_i$. The property $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$ is defined as the set of all graphs having a vertex $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-partition. If a property $\mathcal{R} \in M^a$ can be expressed as the product of at least two properties from M^a, then it is said to be *reducible in* M^a; otherwise it is called *irreducible*. If $\mathcal{P}$ is an induced-hereditary property, then the set of *minimal forbidden subgraphs* of $\mathcal{P}$ is defined as follows: $\boldsymbol{F}(\mathcal{P}) = \big\{G : G \notin \mathcal{P}$ but each proper induced-subgraph H of $G \in \mathcal{P}\big\}$. The property $\mathcal{P}$ is said to be additive if it is closed under disjoint union.

We write $G \stackrel{v}{\longrightarrow} (H)^k$, $k \geq 2$, if for each k-colouring $V_1, V_2, \ldots, V_k$ of a graph G there exists i, $1 \leq i \leq k$, such that the graph induced by the set V_i contains H as a subgraph. A graph G is called $(H)^k-$ *vertex Ramsey minimal* if $G \stackrel{v}{\longrightarrow} (H)^k$, but $G' \stackrel{v}{\not\rightarrow} (H)^k$ for any proper subgraph G' of G.

Every additive induced-hereditary property $\mathcal{P}$ is uniquely determined by the set of connected minimal forbidden subgraphs. For the class $\mathcal{O}^k$ of all k-colourable graphs the set $\boldsymbol{F}(\mathcal{O}^k)$ consists of all $(k+1)$- critical graphs. $\boldsymbol{F}(\mathcal{P})$ may be finite or infinite, a long standing open problem whether the family $\boldsymbol{F}(\mathcal{R})$ may be finite for a reducible property $\mathcal{R}$ was solved by A. Berger. She proved that $\boldsymbol{F}(\mathcal{R})$ is infinite for any additive reducible property.

In our talk we will present several results concerning the structure of reducible graph properties and minimal forbidden subgraphs of the class of $\mathcal{R}$, which implies some results on vertex Ramsey minimal graphs, as well.

Complexity and complicacy questions of graphs representable by polygons

Martin Pergel

Among graphs with geometrical representation (intersection graphs) there are explored graphs representable by polygons in a plane (called CONV-graphs) and, say, graphs representable by polygons inscribed in a circle (PC-graphs).

Denote by $\mathrm{cmp}(G)$ the smallest k such that PC-graph G has PC-representation consisting of k-gons. Analogically denote by $\mathrm{cmp}_2(G)$ analogical k for CONV-representation. We show bounds for cmp and cmp_2, bounds for cmp are asymptotically tight. Then we show, that it is NP-complete to decide, whether $\mathrm{cmp}(G)$ is at most k (even for fixed k). This shows e. g. (for the re-opened problem of complexity of PC-graph recognition) that even if PC-graph recognition is polynomially solvable, it is NP-complete to find an optimal PC-representation.

Observability of some regular graphs

Janka Rudašová

(joint work with Roman Soták)

Observability of a graph G is the minimum k for which the edges of G can be properly coloured with k colours in such a way that colour sets of vertices of G (sets of colours of their incident edges) are pairwise distinct. We determine observability of some regular graphs.

Total irregularity strength of complete graphs

Roman Soták

(joint work with S. Jendrol' and J. Miškuf)

A total edge-irregular k-labelling $\varphi : V \cup E \to \{1, ..., k\}$ of a graph $G = (V, E)$ is a labelling of the vertices and the edges of G in such a way that for any two different edges e, f their weights $w(e)$ and $w(f)$ are distinct. The weight of an edge $e = uv$ is the sum of the labels vertices x and y and the edge e. The minimum k for which there exists a total edge-irregular k-labelling of the graph G is called total irregularity strength of G, $tes(G)$. In this paper we focus on complete graphs, complete bipartite graphs and complete multipartite graphs. Exact value of tes number for the mentioned graphs is shown.

Exponents of t-balanced Cayley maps

L'ubica Staneková

A Cayley map is a Cayley graph embedded in an oriented surface, such that the cyclic order of generators is the same at each vertex. The distribution of inverses of a Cayley map is the involution indicating the position of mutually inverse generators in the cyclic order at a vertex. So called t-balanced Cayley maps have a special (linear) distribution of inverses. Loosely speaking, an exponent of a map is a number e with the property that the Cayley map is isomorphic to its 'e-fold rotational image'.

In our contribution we will present results related to the construction of t-balanced Cayley maps without exponent t which are not regular.

Minimum 4-geodetically connected graphs

Jozef Škorupa

(joint work with Ján Plesník)

A graph G is k-geodetically connected if it is connected and the removal of at least k vertices is required to increase the distance between at least one pair of vertices or reduce G to a single vertex. We define the class of k-plet graphs which for a given number of vertices have the fewest edges of all k-geodetically connected graphs with minimum degree k. We also characterize the class of minimum 4-geodetically connected graphs which have the fewest edges for a given number of vertices.

Antibandwidth and cyclic antibandwidth of meshes and hypercubes

L'ubomír Török

(joint work with André Raspaud, Heiko Schröder, Ondrej Sýkora and Imrich Vrt'o)

The antibandwidth problem consists of placing the vertices of a graph on a line in consecutive integer points in such a way that the minimum difference of adjacent vertices is maximized. The problem was originally introduced in connection with multiprocessor scheduling problems and can be also understood as a dual problem to the well known bandwidth problem, as a special radiocolouring problem or as a variant of obnoxious facility location problem. The antibandwidth problem is NP-hard, there are a few classes of graphs with polynomial time complexities. Exact results for nontrivial graphs are very rare. Miller and Pritikin showed tight bounds for 2-dimensional meshes and hypercubes. We solve the antibandwidth problem precisely for two dimensional meshes, tori and estimate the antibandwidth value for hypercubes up to the third order term. The cyclic antibandwidth problem is to embed an n-vertex graph into the cycle C_n, such that the minimum distance (measured in the cycle) of adjacent vertices is maximized. This is a variant of obnoxious facility location problems or a dual problem to the cyclic bandwidth problem. The problem is NP-hard. We prove basic facts and exact results for meshes, tori and asymptotics for hypercubes.

On maps with the face incident with all vertices

Milan Tuhársky

It is already known that every outerplanar graph contains an edge e with the sum of degrees of its endvertices at most 6. This sum is called the *weight* of e. We investigated similar problem for graphs embeddable in an orientable surface $\mathrm{S_g}$. We prove that every map G on $\mathrm{S_g}$ containing a face incident with all vertices contains an edge of weight depending on the genus g of $\mathrm{S_g}$; particular: Let G be a map on $\mathrm{S_g}$ with the vertex degree at least 2. Then there is an edge e in G of weight at most 10 if G is the map on the torus or an edge of weight at most *4g+5* if G is the map on $\mathrm{S_g}$, $\mathrm{g} \geq 2$.

Radius-invariant graphs

Ondrej Vacek

A graph G is said to be radius-edge-invariant if $r(G-e) = r(G)$ for every $e \in E(G)$, radius-vertex-invariant if $r(G-v) = r(G)$ for every $v \in V(G)$ and radius-adding-invariant if $r(G+e) = r(G)$ for every $e \in E(\overline{G})$.

We present various results concerning radius-invariant graphs.

Decompositions of graphs

Tomáš Vetrík

Decompositions of complete graphs into factors with given diameters are known to have the following hereditary property: If K_n is decomposable into m factors with diameters $d_1, d_2, ..., d_m$, then so is any $K_{n'}$ for $n' > n$. Let $F(d_1, d_2, ..., d_m)$ denote the smallest n for which K_n admite a decomposition into m factors with diameters $d_1, d_2, ..., d_m$. In this contribution we prove that $F(3, d_2, d_3) = d_2 + d_3 - 6$ for any d_2 and d_3 such that $9 \leq d_2 \leq d_3 < \infty$.

Square of metrically regular graphs

Vladimír Vetchý

Let X be a finite set, $n := |X| \geq 2$. For an arbitrary natural number D let $\mathbf{R}= \{R_0, R_1, \ldots, R_D\}$ be a system of binary relations on X. A pair $(X,\mathbf{R})$ will be called *an association scheme* with n classes if and only if it satisfies the axioms $A1 - A4$:

A1. The system $\mathbf{R}$ forms a partition of the set X^2 and R_0 is the diagonal relation, i.e. $R_0 = \{(x, x); x \in X\}$.

A2. For each $i \in \{0, 1, \ldots, D\}$ it holds $R_i^{-1} \in \mathbf{R}$.

A3. For each $i, j, k \in \{0, 1, \ldots, D\}$ it holds
$(x, y) \in R_k \wedge (x_1, y_1) \in R_k \Rightarrow p_{ij}(x, y) = p_{ij}(x_1, y_1)$,
where $p_{ij}(x, y) = |\{z; (x, z) \in R_i \wedge (z, y) \in R_j\}|$.
Then define $p^k_{ij} := p_{ij}(x, y)$ where $(x, y) \in R_k$.

A4. For each $i, j, k \in \{0, 1, \ldots, D\}$ it holds $p^k_{ij} = p^k_{ji}$.

The set X will be called the *carrier* of the association scheme $(X,\mathbf{R})$. Especially, $p^k_{i0} = \delta_{ik}$, $p^0_{ij} = v_i\delta_{ij}$, where δ_{ij} is the Kronecker-Symbol and $v_i := p^0_{ii}$, and define $P_j := (p^k_{ij})$, $0 \leq i, j, k \leq D$.

Given an undirected graph $G = (X, E)$ of diameter D we may now define $R_k = \{(x, y); d(x, y) = k\}$, where $d(x, y)$ is the distance from the vertex x to the vertex y in the standard graph metric. If $(X,\mathbf{R})$ gives rise to an association scheme, the graph G is called *metrically regular* (sometimes also called *distance regular*) and p^k_{ij} are said to be its *parameters*. In particular, a metrically regular graph with diameter $D = 2$ is called *strongly regular*.

Let $G = (X, E)$ be an undirected graph without loops and multiple edges. *The second power* (or *square* of G) is the graph $G^2 = (X, E')$ with the same vertex set X and in which mutually different vertices are adjacent if and only if there is at least one path of the length 1 or 2 in G between them.

The necessary conditions for G to have the square G^2 metrically regular are found and some constructions of those graphs are solved for metrically regular graphs of diameter $D = 3$ and for metrically regular bigraphs of diameter $D = 3, 4, 5, 6$ and 7.

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