GRAFY 2016

51st Czech-Slovak Conference on Graph Theory and Combinatorics

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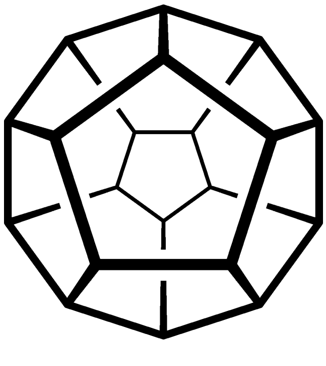
Zborník abstraktov

Abstract booklet



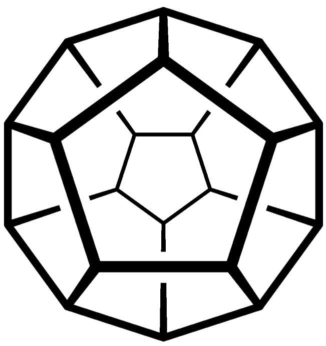
Comenius University, Bratislava
Slovak Mathematical Society

Zborník abstraktov

51. česko-slovenskej konferencie Grafy 2016

Abstract booklet

of the 51st Czech-Slovak conference Grafy 2016



Comenius University, Bratislava
Slovak Mathematical Society

Pozvaní prednášajúci / Invited speakers:

Igor Fabrici
Štefan Gyürki
Jan Hladký
Petr Kolman
Jakub Przybylo
Matěj Stehlík
Jana Šiagiová

Prípravný výbor / Organizing committee:

Robert Jajcay
Robert Lukoťka
Martin Mačaj
Edita Máčajová
Ján Mazák
Martin Škoviera

Conference logo: Marcel Abas

List of talks

Large Cayley graphs of diameter two

Marcel Abas

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In this talk we present a construction of Cayley graphs of diameter two with order $\frac{200}{289}(d+1)^2$ for every degree $d = 17n - 1$, where $n \equiv 1 \pmod{10}$ is a prime.

In addition, using explicit estimates for the distribution of primes in arithmetic progressions, we show that for every degree $d \geq 360756$ there is a Cayley graph of diameter two and of order at least $0.684d^2$.

A central repository of discrete objects

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DiscreteZOO is a project which aims to combine a central repository of discrete objects, its website front-end and extensions for software packages like Sage. At the moment, the repository contains three censuses of symmetric graphs and certain precomputed properties to speed up the processes of filtering, searching and computation. The groundwork is already laid out for more combinatorial objects.

In the talk we will present the structure of the repository and show how one can interact with DiscreteZOO on the example of the census of cubic vertex transitive graphs (by Potočnik, Spiga and Verret).

An alternative representation of symmetries in graphs and networks

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Let the graphs G and H be given. A k-morphism f from G to H is a bijection between the families of k-element subsets of vertices of G and H such that the subgraphs $G - X$ and $H - f(X)$ have the same number of edges. In this contribution, we consider the properties that a k-morphism must have to be the representation of the isomorphism from G to H or the automorphism of G. We show that the automorphism group and the vertex stabilisers of G can be represented by the sets of special k-morphisms. We also consider the possibilities which allow us to use the above mentioned mappings to define generalised symmetries in (asymmetric) graphs, and generalised similarity in complex networks.

L(2,1)-labelling of cacti

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(joint work with Michal Forišek)

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An L(2,1)-labelling is a labelling of the vertex set of graph with non-negative integers such that the labels of adjacent vertices differ by at least two and the labels of vertices at distance 2 are distinct. It is required to determine, for a given graph G, the smallest integer k such that G admits an L(2,1)-labelling with integers not exceeding k; this invariant is denoted by $\lambda(G)$. Determining $\lambda(G)$ is known to be a hard problem. To test whether $\lambda(G) \leq k$ is NP-complete even for series-parallel graphs. On the other hand, there exist classes of graphs where this problem is polynomially solvable (e.g. trees or their mild generalisations). In this talk we derive tight upper and lower bounds for the λ-number of cacti. We also present a polynomial-time algorithm which computes the λ-number of an arbitrary cycle-tree (cactus with disjoint cycles).

Connected even factors in the square of 2-edge-connected graphs

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An essentially k-edge connected graph G is a connected graph such that deleting less than k edges from G cannot result in two nontrivial components. In this talk we prove that if an essentially 2-edge connected graph G satisfies that for any pair of leaves at distance 4 in G there exists another leaf of G that has distance 2 to one of them, then the square G^2 has a connected even factor with maximum degree at most 4. Moreover we show that, in general, the square of essentially 2-edge connected graph does not contain a connected even factor with bounded maximum degree.

The further research will continue with a study of graphs with nontrivial bridges not sharing a vertex in common.

Colorings of plane graphs

IGOR FABRICI

(joint work with Frank Göring, Stanislav Jendroľ, and Michaela Vrbjarová)

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A vertex (edge) coloring $\varphi: V \to \{1, \dots, k\}$ ($\varphi: E \to \{1, \dots, k\}$) of a plane graph $G = (V, E, F)$ is

- *weak parity* (*WP*) if, for every face $\alpha \in F$, there is a color c with the odd number of vertices (edges) of α colored by c,
- *conflict free* (*CF*) if, for every face $\alpha \in F$, there is a color c with the unique vertex (edge) of α colored by c,
- *unique maximum* (*UM*) if, for every face $\alpha \in F$ and the maximum color c used on α, there is the unique vertex (edge) of α colored by c.

In this talk, we give a survey of known results on (proper) WP (CF, UM) colorings of plane graphs.

Trimmed Gray codes

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A *saturating* cycle in a bipartite graph is a cycle covering an entire bipartite class. This naturally generalizes Hamilton cycles for unbalanced bipartite graphs. We show that the subgraph of the n-dimensional hypercube Q_n induced on any k consecutive levels has a saturating cycle for any odd $k \geq 3$ and any $n \geq k$. This is achieved by so called *trimming* of the reflected Gray code in Q_n. For even k, a saturating cycle exists if the first and the last levels are not separated by the middle level(s). Otherwise, the problem is equivalent to the generalized middle level conjecture, which is still open.

In the above cases when we have a saturating cycle, we also provide a (cyclic) enumeration of all vertices from any k consecutive levels of Q_n such that the total distance between consecutive vertices is minimized. Moreover, for odd k we describe a *loopless* algorithm generating any l consecutive vertices (resp. their transitional sequence) of the saturating cycle or the minimal total distance enumeration in time $O(l)$; that is, independently on n.

From strongly regular graphs to directed strongly regular graphs

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(joint work with Mikhail Klin)

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In the talk the concept of strongly regular graphs will be discussed. The undirected version has been known for longer time. We briefly touch several algebraic and combinatorial properties of them, as well as their connections to other parts of mathematics. After that we will switch to directed strongly regular graphs. We will point out which properties are similar to those for undirected case, and which are different in this case. At the end of the talk we will present several new constructions, which result in discovery of infinite families of directed strongly regular graphs.

Tilings in graphons

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In the talk we describe a general theory developed in [4], as well as further properties of this theory which were obtained in [2] and an application contained in [2]. Some of these results were announced in an extended abstract [3].

Much of extremal graph theory concerns with investigating relations between subgraph densities. For example, Turán's Theorem (more precisely, its simplest version due to Mantel) says that any graph with edge density more than $1/2$ must have a positive density of triangles. This was extended famously by Razborov [9] who determined an optimal function $f_3 : [0,1] \to [0,1]$ such that for any n-vertex graph G, if

$$G \text{ contains } \geq \alpha\binom{n}{2} \text{ edges then } G \text{ contains } \geq (f_3(\alpha) \pm o(1))\binom{n}{3} \text{ triangles .} \tag{1}$$

This settled the "triangle-case" of a conjecture of Lovász and Simonovits which had been open for several decades. The functions $f_4, f_5, \ldots$ corresponding to densities of complete graphs of order $4, 5, \ldots$ were obtained by Reiher [10]. The key in obtaining these results was the theory of flag algebras developed shortly before that by Razborov in [8]. The theory of flag algebras as well as the closely related theory of limits of dense graph sequences (see e.g. [7]) have led to a number of further breakthroughs in extremal graph theory. While variants are possible, many of these results follow the same pattern as (1), namely

$$\text{density} \geq a \text{ of a graph } F \text{ guarantees density} \geq b + o(1) \text{ of a graph } H$$

(all in an ambient large graph G; the error term $o(1)$ refers to the order of G).

Here, we consider "tiling" counterparts to such problems. Our inspiration comes for example from recent work of Allen, Böttcher, Hladký and Piguet [1] who determined an optimal function $g_3 : [0,1] \to [0,1]$ such that for any n-vertex graph G, if

$$G \text{ contains } \geq \alpha\binom{n}{2} \text{ edges then } G \text{ contains } \geq (g_3(\alpha) \pm o(1))n \text{ vertex-disjoint triangles.} \tag{2}$$

So, the difference between (1) and (2) is that the assertion in the former is in terms of a density of a fixed graph while in the latter it is in the size of a maximum family of vertex-disjoint copies of that graph (i.e., the triangle).

More generally, we say that a family of (not necessarily induced) copies of F in a graph G is an *F-tiling*. Then the *F-tiling number* is just the size of a largest such family.

Our results

We translate the notion of F-tiling and of F-tiling number (and their fractional variants) to the setting of graphons (which are limits of finite graphs). We also translate the dual notion of (fractional) F-covers. The case $F = K_2$ gives the notion of matchings in graphons. We give a transference statement that allows us to switch between the finite and limit notion, and derive several favorable properties, including the LP-duality counterpart to the classical relation between the fractional vertex covers and fractional matchings/tilings, and discuss connections with property testing.

We give two applications of this theory. Firstly, we determine the asymptotically almost sure F-tiling number of inhomogeneous random graphs $\mathbb{G}(n, W)$. Secondly, we give a short proof of graphon version of a tiling theorem of Komlós [6] which in this context extends the Hajnal–Szemerédi Theorem. This version of Komlós's theorem together with the transference result implies the original result. Actually, we strengthen Komlós's Theorem in that we also prove its stability version.

Paralleling the main object of study in combinatorial optimization, we study "matching polyton" and "vertex-cover polytons", which are graphon counterparts to matching polytopes and vertex cover polytopes studied in finite graphs.

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Möbius regular maps on the group PSL$(2, q)$

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Maps are cellular embeddings of graphs into closed compact surfaces. Regular maps are maps with the highest possible level of symmetry, which means that the automorphism group of a regular map acts regularly on the set of its flags.

We will focus on a special class of regular maps – so called Möbius regular maps, i.e. regular maps in which any two distinct adjacent vertices are joined by exactly two edges and any open set on the supporting surface of a map containing these two edges contains a Möbius band.

We will derive a condition for a regular map with automorphism group PSL$(2, q)$ to be Möbius, and based on it we will examine when such object does exist.

On external symmetries of Wilson's maps

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It is well known that a regular map M can by identified with a presentation of its automorphism group generated by three involutions, two of which commute. In the talk we will consider the family of regular maps M_k given by the following presentations of their automorphism groups: $G_k = \langle a, b, c \mid a^2 = b^2 = c^2 = (ac)^2 = (ab)^{2k} = (cb)^{2k} = (acb)^{2k} = (abacbcb)^2 = 1\rangle$. This family, introduced by S. Wilson in 1976, is remarkable as its members are self-dual, self-Petrie-dual and admit all feasible exponents [1]. The group these dualities and exponents generate (of order $6 \cdot (\varphi(2k))^3/m$ where m is the number of units r in Z_2k that are solutions of $r^2 \equiv 1 \bmod k$) was determined in [2] and it is a subgroup of the automorphism group of G_k.

Any composition of the two dualities and exponents may be called an 'external symmetry' of a regular map. One may, however, ask whether such symmetries might be introduced without reference to dualities and exponents. An option is to stipulate that an external symmetry of a regular map M with automorphism group $= \langle a, b, c \mid a^2 = b^2 = c^2 = (ac)^2 = \ldots = 1\rangle$ be an automorphism of G that preserves the subgroup $\langle a, c\rangle$ whose cosets in G represent edges of M. We determined this group for the family of Wilson's maps M_k and it turns out that its order is $6 \cdot (\varphi(2k))^3$. This has the surprising consequence that there are external symmetries (in the above sense) that do not arise as compositions of dualities and exponents operators.

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Combinatorial structures with regular automorphism groups

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The concept of the automorphism group of a combinatorial structure is a fundamental concept both in Combinatorics and Permutation Group Theory. Finding the automorphism groups of combinatorial structures is generally a hard problem whose exact complexity is not known. The main topic of the talk is the opposite problem of constructing a combinatorial structure for a given automorphism group.

The left regular action of a group on itself is one of the most natural group actions. The aim of our presentation is constructing combinatorial structures whose full automorphism groups act regularly on their sets of vertices. Equivalently, we attempt to classify finite groups which admit the introduction of a combinatorial structure whose full automorphism group consists solely of the automorphisms induced by the multiplication by the elements of the underlying group. Such structures can be thought of as combinatorial representations of the corresponding groups.

Previous results on this topic include the classification of graphical regular representations (graphs with regular automorphism groups), classification of digraphical regular representations (directed graphs with regular automorphism groups), as well as the classification of general combinatorial structures (incidence structures) with regular automorphism groups. We generalize these results to the class of k-hypergraphs which are incidence structures with all blocks of size k, and consider the spectrum of all k's for which such representation is possible.

Homogeneous colouring

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An k-homogeneous colouring of a graph G is a proper colouring of vertices of G such that the number of colours in the neighbourhood of any vertex equals k. We explore properties of such colouring in general as well as for regular and other particular graphs.

Algorithms for regular 3-edge-colouring of subcubic graphs

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Snark is cyclically 4-connected cubic graph of girth at least 5 without (a regular) 3-edge-colouring. A motivation to study snarks comes from the fact that many important and difficult problems of graph theory and discrete mathematics can be formulated as questions on snarks. In present, computer-aided experiments are used in investigation of snarks mainly. Among others, all snarks up to 36 vertices were constructed. On the other hand, the structural properties of (constructed) snarks are not known adequately. The most of tests for chosen properties (invariants) of a snark G employs the test for the existence of 3-edge-colouring of some subgraph of G.

It transpires that effectivity and real usefulness of computers for solving of these problems depends on effectiveness of the algorithm (and its implementation) for testing the 3-edge-colourability of a subcubic graph. However, the problem of 3-edge-colourability of cubic graphs is NP-complete. Almost all algorithms implemented so far are based on back-track. Recently, several alternative approaches based on heuristics and randomisation appeared in literature. We will discuss them briefly and we will show how they are employed in various tests for certain important invariants of snarks. We will mention also modern approaches in implementation, including parallel and distributed computations.

Join products and Cartesian products crossing numbers

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The *crossing number* $\mathrm{cr}(G)$ of a graph G is the smallest number of edge crossings in any drawing of G in the plane. It was introduced by Turán during the World War II. The crossing number problem has been vividly investigated for over 60 years, but there are only few infinite graph classes for which the values of crossing number are known. Even for the complete and complete bipartite graphs this invariant is known only for some few parameters. It was shown by D.J. Kleitman that the crossing number of the complete bipartite graph $K_{m,n}$ is $\lfloor \frac{m}{2} \rfloor \lfloor \frac{m-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ for all $m \leq 6$ and all n. In general, computing the crossing number is very difficult. In 1973, Harary, Kainen, and Schwenk established the crossing number of the Cartesian product $C_3 \times C_3$ and conjectured that $\mathrm{cr}(C_m \times C_n) = m(n-2)$ for $3 \leq m \leq n$. Recently has been proved by Glebsky and Salazar that for any fixed $m \geq 3$, the conjecture holds for all $n \geq m(m+1)$. The crossing numbers of some classes of graphs have been obtained. In 2007, Bokal introduced the Zip-product and proved the conjecture given by Jendroľ and Ščerbová that $\mathrm{cr}(K_{1,n} \times P_m) = (m-1)\lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ for the path P_m of length m.

Let G and H be two disjoint graphs. The *join product* of G and H, denoted by $G + H$, is obtained from vertex–disjoint copies of G and H by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G + H$ is the union of disjoint edge sets of the graphs G, H, and the complete bipartite graph $K_{m,n}$. The Kleitman's result enables us to establish the crossing numbers of several join products.

In the talk, we summarise new results concerning crossing numbers of join product of two graphs. These results, together with Bokal's Zip-product, can be used by establishing the crossing numbers of Cartesian products of special graphs with trees and other graphs.

Graphs with the minimum value of Balaban index

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We consider graphs of order n with the minimum value of Balaban index. First we show that Balaban index is at least $8/n$. Then, for small values of n we determine the extremal graphs. Finally, we show that balanced dumbbell graphs with clique sizes $\sqrt[4]{\pi/2}\sqrt{n} + o(\sqrt{n})$ have the value of Balaban index about $10.15/n$.

From Length Bounded Cut Problem to Extended Formulations for MSOL Problems on Graphs of Bounded Treewidth

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Given a graph $G = (V, E)$ with two distinguished vertices $s, t \in V$ and an integer parameter L, an *L-bounded cut* is a subset F of edges such that every path between s and t in $(V, E \setminus F)$ has length at least $L + 1$. The *minimum L-bounded cut* problem is to find an L-bounded cut of minimum cardinality. In the first part of the presentation we will survey known results about this simple to state yet challenging problem.

Then we move another problem whose study was motivated by a search for better approximation algorithms for the L-bounded cut problem. Given a graph $G = (V, E)$ and an MSOL formula φ, we consider the *convex hull $P_\varphi(G)$ of all satisfying assignments* of φ. We sketch a proof showing that there exists an extended formulation of the polytope $P_\varphi(G)$ that can be described by $f(|\varphi|, \tau) \cdot n$ inequalities, where $n = |V|$, τ is the treewidth of G and f is a computable function depending only on φ and τ.

The minimization of the maximal out-degree in a simple graph

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(joint work with Petr Kovar, Adam Silber, and Michal Kravcenko)

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In this talk we discuss the minimization of the maximal out-degree in a general simple graph G. First, we talk about the motivation for this problem and we mention known results.

We extend these known results by defining two new algorithms. We introduce a linear-time algorithm for r-regular graphs, based on finding a set of eulerian paths. Furthermore, we introduce a heuristic algorithm, which gives good results for many graphs. Finally, we show the construction of a graph for which this heuristic algoritm does not work correctly.

Factorizations of complete graphs into tadpoles

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(joint work with Tom Raiman)

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We investigate factorizations of complete graphs K_{2n+1} into isomorphic tadpoles for $n \geq 2$. A *tadpole* is a unicyclic graph G that arises from a cycle C_m and a path of length $|V(G)| - m$ so that we glue a terminal vertex of a path to an arbitrary vertex of a cycle. We show that every tadpole on $4k+3$ vertices factorizes K_{4k+3}. On the other hand the research of factorizations of K_{4k+1} into tadpoles is not finished yet.

Families of plane graphs with prescribed δ, ρ, w and w^*

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(joint work with Peter Hudák, Tomáš Madaras, Pavol Široczki)

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Let $G = (V, E, F)$ be a connected plane graph. The minimum vertex degree of G we denote by $\delta(G)$. For an edge $e = xy$ the weight of an edge e is the sum $w(e) = deg(x) + deg(y)$, and the minimum edge weight of G we denote by $w(G)$. The minimum face size of G we denote by $\rho(G)$ and the minimum dual edge weight of G is the number $w^*(G) = \min\{d(\alpha) + d(\beta) : \ \alpha, \beta \in F, \ \alpha \neq \beta, \ \alpha, \beta \text{ have a common edge}\}$.

We study the families of plane graphs with $\delta(G) = 2$ determined by lower bounds ρ, w, w^* on their face sizes, edge weights and dual edge weights, respectively. Continuing the previous research of families of polyhedral graphs ($\delta(G) \geq 3$), we determine the quadruples $(2, \rho, w, w^*)$ for which the associated family is non-empty. In addition, we determine all quadruples which yield extremal families (in the sense that the increase of any value of a quadruple results in empty family of graphs).

On improper interval edge colourings

Tomáš Madaras

(joint work with Peter Hudák, František Kardoš, Michaela Vrbjarová)

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An edge colouring is called *interval colouring* if edge colours around each vertex form an integer interval. Proper interval colourings were introduced by Asratian and Kamalian in 1987 and, since then, were extensively studied. In our contribution, we relax the condition of interval colouring to be proper and study the value $\widehat{t}(G)$ being the maximum number of colours in an improper interval colouring of a graph G; we present several general estimates of $\widehat{t}(G)$ as well as exact values and estimates for graphs of particular families, namely wheels, prisms and complete graphs.

On the cold cases in incidence coloring

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An *incidence* in a graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are adjacent if at least one of the following holds: (i) $v = u$, (ii) $e = f$, or (iii) vu is an edge from the set $\{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The corresponding chromatic number is called *incidence chromatic number*.

In this talk we present some results on graphs regarding their maximum degree and maximum average degree. It is known that every planar graph G with maximum average degree $\mathrm{mad}(G) < 3$ and $\Delta(G) \geq 5$ has the incidence chromatic number at most $\Delta(G) + 2$. We obtain the same bound for such graphs with $\Delta(G) = 4$. Moreover, for graphs with maximum average degree $\mathrm{mad}(G) < \frac{10}{3}$ and $\Delta(G) \geq 8$ we show that the incidence chromatic number is at most $\Delta(G) + 2$.

It was also proved that at most $\Delta(G) + 5$ colors are enough for an incidence coloring of any planar graph G except $\Delta(G) = 6$; in this case at most 12 colors are needed. We improve the bound for $\Delta(G) = 6$ to 10.

Vertex-partitions of cubic graphs and cyclic connectivity

Roman Nedela

(joint work with M. Škoviera)

We show that if a cubic graph X is cyclically 4-connected and the Betti number is even, then it admits a partition into an induced tree and an independent set. If the Betti number is odd, then one can guarantee one of the two kinds of partitions: Either $V(X)$ decomposes into an induced tree and near-independent set of vertices, or it decomposed into an induced forest with two components and independent set. We prove that if X is cyclically 5-connected with an odd Betti number, then a decomposition of the first type exists.

To prove the latter statements one needs to characterise cyclically 5-connected cubic graphs which are critical in the following sense: Removal of every pair of adjacent vertices decreases the cyclic connectivity by two. In fact the only such graph is the Petersen graph.

The inertia and D-energy of distance matrices of some H-join graphs

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(joint work with Pavel Híc)

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Let D denote the distance matrix of a connected graph G. The inertia of D is the triple of integers $(n_+(D), n_0(D), n_-(D))$, where $n_+(D), n_0(D)$ and $n_-(D)$ denote the number of positive, 0, and negative eigenvalues of D, respectively. The D-energy is the sum of the absolute values of eigenvalues of D.

Since the distance matrix of a general graph is complicated, it is very difficult to compute its inertia. Therefore, the inertia of the distance matrices of some special classes of graphs is studied.

Up to now, only few classes of graphs which have exactly one positive D-eigenvalue, are known. Such are for example complete graphs, trees, connected unicyclic graphs, the polyacenes, honeycomb and square lattices, iterated line graphs of some regular graphs. Recently, new classes of graphs with exactly one positive D-eigenvalue such as cacti graphs, wheel graphs, clique trees were constructed. This inspired us to get more graphs with exactly one positive D-eigenvalue.

In this contribution, we study the inertia of distance matrices of H-join of distance regular graphs of diameter at most 2. Then, as application, we give the inertia and D-energy of graphs of the form $\vee_{K_n}(K_{2,...,2}, K_{2,...,2}, ..., K_{2,...,2})$ and $\vee_{K_{1,k}}(K_2, K_{n-2}, ..., K_{n-2})$. These graphs have exactly one positive D-eigenvalue. Moreover, by iteration of H-join on the first of above graphs and modification of the second graph we gain further classes of graphs with exactly one positive eigenvalue.

Asymptotically optimal neighbour distinguishing edge colourings of graphs

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We shall discuss a few important problems from the field of *additive graph labellings*, or more generally – *vertex distinguishing graph colourings*, stemming form research on irregularities in graphs. As the key example, the following problem shall be most thoroughly discussed.

Consider a simple graph $G = (V, E)$ and its *proper* edge colouring c with the elements of the set $\{1, 2, \ldots, k\}$. The colouring c is said to be *neighbour sum distinguishing* if for every pair of vertices u, v adjacent in G, the sum of colours of the edges incident with u is distinct from the corresponding sum for v. The least integer k for which such colouring exists is known as the *neighbour sum distinguishing index* of a graph and denoted by $\chi'_{\sum}(G)$. The definition of this parameter, which makes sense for graphs containing no isolated edges, immediately implies that $\chi'_{\sum}(G) \geq \Delta$, where Δ is the maximum degree of G. On the other hand, it was conjectured by Flandrin et al. that $\chi'_{\sum}(G) \leq \Delta + 2$ for all such connected graphs, except for C_5. During the talk, a proof of the fact that this bound is asymptotically correct, i.e. that $\chi'_{\sum}(G) \leq \Delta(1 + o(1))$ shall be presented. The main idea of the argument relays on a random assignment of the colours, where the choice for every edge is biased by so called *attractors*, randomly assigned to the vertices.

Flows on signed graphs with two negative edges

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Bouchet's conjecture states that every flow-admissible signed graph, a graph where each edge has a positive or negative sign, admits a nowhere-zero 6-flow. The conjecture of Bouchet is true for signed graphs with 0 negative edges by the 6-flow theorem of Seymour for unsigned graphs. In addition, signed graphs with 1 negative edge are not flow-admissible. Here we investigate signed graphs with 2 negative edges, and prove that every such flow-admissible graph has a nowhere-zero 7-flow. Furthermore, if Tutte's 5-flow conjecture for graphs is true, Bouchet's conjecture holds true for signed graphs with 2 negative edges.

The achromatic and Grundy numbers of Kneser graphs

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Let V be the set of all k-subsets of $\{1, 2, \ldots, n\}$, where $1 \leq k \leq n/2$. The *Kneser graph* $K(n, k)$ is the graph with vertex set V such that two vertices are adjacent if and only if the corresponding subsets are disjoint.

The *achromatic number* α of a graph is the largest number of colors for which a proper coloring has the property that each pair of distinct colors are on the end-vertices of some edge. The *Grundy number* $\Gamma(G)$ is the largest number of colors for which a proper coloring has the property that for every two colors i and j, with $i < j$, every vertex colored j has a neighbor colored i.

We study the achromatic and the Grundy numbers of Kneser graphs $K(n, k)$ and estimate $\alpha(K(n, k))$ and $\Gamma(K(n, k))$ for some values of n and k.

Minimal unavoidable sets of cycles in polyhedral graphs

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A set S of cycles is minimal unavoidable in a graph family $\mathcal{G}$ if each graph $G \in \mathcal{G}$ contains a cycle from S and, for each proper subset $S' \subset S$, there exists an infinite subfamily $\mathcal{G}' \subseteq \mathcal{G}$ such that no graph from $\mathcal{G}'$ contains a cycle from S'. We explore unavoidable sets of cycles in the family of polyhedral graphs. It is well known that the set $S_{3,4,5}$ consisting of cycles of lengths 3, 4 and 5 is minimal unavoidable in this family; we show, in addition, that the sets $S_{4,6,8,10}$ and $S_{5,6,7,11}$ are also minimal unavoidable, and we present several constructions of polyhedral graphs which show that, for certain ranges of lengths, no set of cycles is unavoidable in polyhedral graphs.

A survey on Cayley graphs in the degree-diameter problem

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The degree-diameter problem is to determine, for given d and k, the largest order $n(d,k)$ of a graph of maximum degree d and diameter k. Since a number of constructions of 'large' graphs for the degree-diameter problem are Cayley, it is of interest to study the largest order $Cay(d,k)$ of a Cayley graph of degree d and diameter k. In the talk we survey the available results on the parameter $Cay(d,k)$.

Antimagic labelings of trees with vertices of degree two only on paths of a fixed length

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Antimagic labelings of graphs is a classical open problem of graph labelings. There is a conjecture (Hartsfield and Ringel) that every tree except K2 is antimagic, but the proof is still missing.

By an antimagic labelling of $G = (V, E)$ we understand a vertex antimagic edge labeling λ as a bijection from the set of edges to $\{1, 2, \ldots, |E(G)|\}$ such that the sum of the labels of incident edges is unique for each vertex.

In the talk we show some known results for trees without the vertices of degree two and present an example on how these results can be used for labeling a very specific but infinite class of trees with vertices of degree two.

On incidence colorings of graphs

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An *incidence* in a graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are *adjacent* if at least one of the following holds: (a) $v = u$, (b) $e = f$, or (c) $vu \in \{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The originators conjectured that every graph G admits an incidence coloring with at most $\Delta(G) + 2$ colors. The conjecture is false in general, but there are many classes of graphs for which it holds, e.g. subcubic graphs, hypercubes, etc. While there exists a graph with maximum degree 6, which needs 9 colors, it is still an open question if the conjecture is true for graphs with maximum degrees 4 and 5. In the talk we will prove a weaker bound that every subquartic graph admits an incidence coloring with at most 7 colors.

Topological bounds on the chromatic number

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Topological bounds on the chromatic number of graphs originate from Lovász's celebrated proof of Kneser's conjecture in 1977. The general idea is to associate a simplicial complex to a graph, and then to bound the chromatic number of the graph in terms of certain topological invariants of the associated complex.

In this talk, we will explore an alternative approach using what we call higher-dimensional projective quadrangulations. We will illustrate this on some classes of graphs such as Kneser graphs, and also show how one of the 'classical' topological bounds can be expressed purely in terms of projective quadrangulations.

(R,M)-WORM colorings of plane graphs

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Given three planar graphs R, M, and G. An (R, M)-WORM coloring of G is a vertex coloring such that no subgraph isomorphic to R is rainbow and no subgraph isomorphic to M is monochromatic. If G has at least one (R, M)-WORM coloring, then $W^{-}_{R,M}(G)$ denotes the minimum number of colors in an (R, M)-WORM coloring of G. We show that:

a) $W^{-}_{R,M}(G) \leq 2$ if $|V(F)| \geq 3$ and H contains a cycle,
b) $W^{-}_{R,M}(G) \leq 3$ if $|V(F)| \geq 4$ and H is a forest with $\Delta(H) \geq 3$,
c) $W^{-}_{R,M}(G) \leq 4$ if $|V(F)| \geq 5$ and H is a forest with $1 \leq \Delta(H) \leq 2$.

The cases when both R and M are nontrivial paths are more complicated; therefore we consider a relaxation of the original problem. Among others, we prove that any 3-connected plane graph (resp. outerplane graph) admits a 2-coloring such that no facial path on five (resp. four) vertices is monochromatic.

On hamiltonicity of $\{K_{1,4}, K_{1,4}+e, S(2,1,1)\}$-free graphs

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Let $S(2,1,1)$ denote the graph obtained from the claw $K_{1,3}$ by subdividing one edge. Kaiser and Vrána recently proved that every 5-connected claw-free graph with minimum degree at least 6 is hamiltonian. In the talk, we extend this result to the class of 5-connected $\{K_{1,4}, K_{1,4}+e, S(2,1,1)\}$-free graphs with minimum degree at least 6.