

Cryptology – introduction

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Cryptology 1 (2023/24)

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Introduction

- security requirements

Encryption

- simple examples, perfect secrecy, Vernam cipher

- modern symmetric ciphers

- asymmetric ciphers

- attack scenarios, Kerckhoffs's principle, key length

Other topics

- data authenticity, NVD, ...

Introduction

- ▶ Cryptology = cryptography & cryptanalysis
- ▶ security in the presence of an adversary
 - ▶ security means .../ adversary means ...?
 - ▶ the answers take you to different topics in cryptology
- ▶ cryptography: constructions (algorithms, schemes, protocols) for various security requirements:
 - ▶ confidentiality (encryption)
 - ▶ integrity/authenticity (hash functions, message authentication codes, digital signatures)
 - ▶ authentication (protocols)
 - ▶ non-repudiation (digital signatures)
 - ▶ privacy, anonymity, etc.
- ▶ cryptanalysis: breaking or finding weaknesses in cryptographic constructions

Security requirements/goals

- ▶ **confidentiality** – Preserving authorized restrictions on information access and disclosure, including means for protecting personal privacy and proprietary information.
- ▶ **integrity** – Guarding against improper information modification or destruction, and includes ensuring information non-repudiation and authenticity.
- ▶ **authenticity** – The property of being genuine and being able to be verified and trusted; confidence in the validity of a transmission, message, or message originator.
- ▶ **non-repudiation** – Protection against an individual who falsely denies having performed a certain action and provides the capability to determine whether an individual took a certain action, such as creating information, sending a message, approving information, or receiving a message.

source: NIST SP 800-53 Rev. 5, 2020

Cryptology vs. information security

cryptography \subsetneq information security

- ▶ cryptography is not answer to all security needs:
 - ▶ availability (redundancy)
 - ▶ secure software (software engineering, security testing), etc.
- ▶ cryptography is usually not the entire answer to security needs:
 - ▶ cryptography is useless without other security measures
 - ▶ key management, access control, risk assessment, personnel security, information classification, etc.

Constructions

- ▶ Commonly used cryptographic constructions:
 - ▶ encryption, digital signatures, authentication and key distribution protocols, message authentication codes
- ▶ Less common constructions:
 - ▶ secret sharing, onion routing, voting protocols, electronic money etc.
- ▶ “Exotic” constructions:
 - ▶ private information retrieval, searchable encryption, fully homomorphic encryption, etc.

Encryption

- ▶ traditional use of cryptography
- ▶ intuitively, we know what encryption is
 - ▶ fiction ~ E.A. Poe, A.C. Doyle, J. Verne ...
- ▶ encryption provides confidentiality of
 - ▶ communicated data, e.g. SSL/TLS, WPA2/WPA3, S/MIME – preventing data compromise when an attacker eavesdrops
 - ▶ stored data, e.g. BitLocker, VeraCrypt, FileVault 2 – preventing data compromise after the attacker, for example, stole a disk
- ▶ informally:
encryption + decryption ~ encryption scheme ~ cipher

Encryption – terminology 1

- ▶ original data = plaintext
- ▶ data after encryption = ciphertext
- ▶ P , C , K – finite sets of all plaintexts, ciphertexts, keys
- ▶ symmetric/secret key encryption scheme:
 - ▶ key generation (usually random bit string)
 - ▶ encryption: $E : K \times P \rightarrow C$
sometimes randomized $E : K \times R \times P \rightarrow C$
 - ▶ decryption: $D : K \times C \rightarrow P$

Encryption – terminology 2

- ▶ correctness: $\forall k \in K \forall p \in P : D_k(E_k(p)) = p$
for randomized encryption: $\forall k \in K \forall r \in R \forall p \in P : D_k(E_k(r, p)) = p$
- ▶ sometimes more complicated by using various modes of encryption, ...
- ▶ efficiency: no one wants to wait for en/de-cryption
- ▶ security is much more difficult
 - ▶ usually “resistance to all known attacks”
 - ▶ e.g. identity is correct and efficient but insecure

Example – Shift cipher (Caesar cipher) 1

- ▶ alphabet $A = \{A, B, \dots, Z\}$
- ▶ natural mapping between characters and numbers:
 $A \leftrightarrow 0, B \leftrightarrow 1, \dots, Z \leftrightarrow 25$
- ▶ $P = C = A, K = \mathbb{Z}_{26}$
- ▶ encryption: $E_k(p) = p + k \bmod 26$
- ▶ decryption: $D_k(c) = c - k \bmod 26$
- ▶ correctness follows from using inverse operation in decryption;
 $(\mathbb{Z}_{26}, +)$ is a group

Example – Shift cipher (Caesar cipher) 2

- ▶ plaintext longer than single character?
 - ▶ using cipher in a “mode”, e.g. encrypt each individual character separately
- ▶ Julius Caesar used $k = -3$ in his private correspondence
- ▶ security: none (when encrypting natural language text of reasonable length)
 - ▶ small key space, only 26 keys (all keys can be tested)
 - ▶ How easy is to recognize a plaintext?

Example – Simple substitution cipher

- ▶ alphabet $A = \{A, B, \dots, Z\}$
- ▶ $P = C = A$, $K = \{\pi \mid \pi \text{ is a permutation on } A\}$
- ▶ encryption: $E_\pi(p) = \pi(p)$
- ▶ decryption: $D_\pi(c) = \pi^{-1}(c)$
- ▶ trivially correct, long plaintext – each character encrypted individually
- ▶ large number of keys: $|K| = 26! \approx 2^{88.38}$
- ▶ easily broken by frequency and/or pattern analysis (for details see the lecture on cryptanalysis of classical ciphers)
- ▶ E.A. Poe: The Gold-Bug (1843)
 - ▶ steganography (invisible ink, revealed by heating)
 - ▶ includes SSC and a detailed description of its cryptanalysis
- ▶ real-world example: Chilean drug traffickers (2010)
- ▶ many variants: polyalphabetic subst. (multiple substitutions), homophonic subst. (e.g. frequent letters to multiple targets), ...

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Example – Permutation cipher

- ▶ $P = C = A^n$, $K = \{\pi \mid \pi \text{ is a permutation on } \mathbb{Z}_n\}$
- ▶ encryption: $E_\pi(p_0 p_1 \dots p_{n-1}) = p_{\pi(0)} p_{\pi(1)} \dots p_{\pi(n-1)}$
- ▶ decryption: $D_\pi(c_0 c_1 \dots c_{n-1}) = c_{\pi^{-1}(0)} c_{\pi^{-1}(1)} \dots c_{\pi^{-1}(n-1)}$
- ▶ trivially correct
- ▶ long plaintext divided into blocks of length n
- ▶ key space size: $|K| = n!$
- ▶ cryptanalysis: frequency analysis of digrams/trigrams for various key lengths and permutation parts
- ▶ many variants of permutation ciphers

Example – Fleissner/Cardano Grille

- ▶ $2n \times 2n$ square with n^2 perforations (exactly one position chosen for perforation from each quadruple of rotational-symmetric positions)
- ▶ encryption: using perforations to write the plaintext (four times rotating the square by 90°)
- ▶ decryption: rotate the square and read the text
- ▶ key: positions of perforations, i.e. key space size 4^{n^2}
- ▶ long plaintext divided into blocks of length $4n^2$

Example – Fleissner/Cardano Grille 2

- ▶ German army in WWI (withdrawn after 4 months)
- ▶ J. Verne: Mathias Sandorf (1885)

	H		A		T
				E	
		M			
	U			S	
					T
			M		

		A			K
			E		
A					M
		A		N	
	P				R

		O			
D					
	U			C	
			T		
	I				
V		E		O	

T				H	
	E		R		
W					I
		S			
E			O		

Hate must make a man productive.
Otherwise one might as well love.

Karl Kraus

T	H	O	A	H	T
D	E	A	R	E	K
W	U	M	E	C	I
A	U	S	T	S	M
E	I	A	O	N	T
V	P	E	M	O	R

Security of encryption scheme

- ▶ How to define the security of encryption scheme?
What is the goal of an attacker?
 - ▶ find the key ...what about identity?
 - ▶ find plaintext from ciphertext ...what about half of the plaintext?
 - ▶ find at least one bit/character of the plaintext from ciphertext ...
 - ▶ compute any nontrivial function of plaintext?
- ▶ robust security definitions are nontrivial tasks

Perfect secrecy (intuitively)

- ▶ (a priori) probability distribution of plaintexts
 - ▶ e.g. “tomorrow” is more probable than “mjuuwerq”
- ▶ (a posteriori) probability distribution of plaintexts (knowing a ciphertext)
- ▶ def: encryption scheme satisfies perfect secrecy if
 - ‘a priori’ distribution of plaintexts is equal to ‘a posteriori’ distribution of plaintext (i.e. knowing a ciphertext does not change the probability distribution of the plaintexts)*
- ▶ alternative:
 - the probability of obtaining particular ciphertext c for arbitrary plaintext is constant*
- ▶ eavesdropper learns *nothing* from the ciphertext
- ▶ single ciphertext; what about the length of the plaintext?

Vernam cipher (one-time pad) 1

- ▶ $P = C = K = \{0, 1\}^n$, for integer n
- ▶ encryption: $E_k(p) = p \oplus k$, where \oplus denotes bit-wise XOR
- ▶ decryption: $D_k(c) = c \oplus k$
- ▶ correctness: $D_k(E_k(p)) = (p \oplus k) \oplus k = p \oplus (k \oplus k) = p$
- ▶ perfect secrecy if:
 1. keys are random with uniform distribution
 2. keys are not reused (new key is generated for each plaintext)
- ▶ given ciphertext c , can p' be the corresponding plaintext?
 - ▶ sure, if $k' = c \oplus p'$ is used as the key

Vernam cipher (one-time pad) 2

- ▶ keys with nonuniform distribution:
 - ▶ change the probability distribution of plaintexts (after observing ciphertext)
- ▶ reused keys:
 - ▶ let $c_1 = p_1 \oplus k$, $c_2 = p_2 \oplus k$
 - ▶ then $c_1 \oplus c_2 = p_1 \oplus p_2$ (XOR of two plaintexts)
 - ▶ can easily be solved for natural (and other) language texts
...two time pad problem
- ▶ disadvantage: $|\text{key}| = |\text{plaintext}|$
 - ▶ distribution in advance (e.g. DVD)
 - ▶ shorter key \Rightarrow sacrifice of perfect secrecy

Modern symmetric ciphers

- ▶ designed for efficient hardware and software implementations
- ▶ cannot have the perfect secrecy property (usually $|key| < |plaintext|$)
- ▶ operate on bit vectors
- ▶ block ciphers:
 - ▶ encryption/decryption algorithm defined over bit vectors
 - ▶ $E, D : \{0, 1\}^n \times \{0, 1\}^k \rightarrow \{0, 1\}^n$
 - ▶ AES (block size: 128 bits, key size: 128/192/256 bits),
3DES (block size: 64 bits, key size: 112/168 bits)
- ▶ stream ciphers:
 - ▶ key and an initialization vector
 - ▶ finite state deterministic generator producing (pseudo-random) keystream
 - ▶ examples: Snow 3G (LTE networks), ChaCha20 (option in SSL/TLS)
 - ▶ block ciphers in specific modes of operation

Asymmetric encryption schemes

- ▶ each user generates his/her own instance
- ▶ based on intractable mathematical problems
 - ▶ factoring, discrete logarithm, etc.
- ▶ three algorithms (Gen, Enc, Dec):
 - ▶ Gen: public key pk , secret (private) key sk
 - ▶ encryption: $Enc_{pk}(m) = c$
 - ▶ decryption: $Dec_{sk}(c) = m$
- ▶ public key for encryption (everyone can encrypt)
- ▶ secret key for decryption (only the owner can decrypt)
- ▶ correctness:

$$\forall(pk, sk) \leftarrow Gen() \forall m : Dec_{sk}(Enc_{pk}(m)) = m$$

Kerckhoffs's principle

A cryptosystem should be secure even if everything about the system, except the key, is public knowledge.

- ▶ don't rely on secret algorithms
- ▶ replacing (HW or SW) implementation is costly/impossible
- ▶ some known failures:
 - ▶ RC4 stream cipher ...source code leaked on Internet
 - ▶ A5/1 ...algorithm reverse engineered
 - ▶ CSS (Content Scramble System) encryption of DVDs ...reverse engineered
- ▶ protecting the design of cryptosystem is sometimes used (but again, the security should not depend on it)

Attack scenarios – ciphers

- ▶ COA – Ciphertext only attack
 - ▶ attacker gets some ciphertexts
 - ▶ eavesdropping, theft ...
- ▶ KPA – Known plaintext attack
 - ▶ attacker knows plaintext-ciphertext pairs
 - ▶ known headers, data structures, closing sentences ...
- ▶ CPA – Chosen plaintext attack
 - ▶ attacker can (adaptively) choose plaintexts and obtain their encryption
 - ▶ always possible with asymmetric schemes
- ▶ CCA – Chosen ciphertext attack
 - ▶ attacker can (adaptively) choose ciphertexts and obtain their decryption

We know neither the environment nor the operational conditions of an encryption scheme \Rightarrow use the strongest possible scheme (with respect to an attack scenario).

Attack scenarios vs. simple ciphers

- ▶ Simple substitution cipher
- ▶ COA: frequency/patterns analysis
- ▶ KPA: reveals π values for all symbols appearing in the plaintext
- ▶ CPA: chosen plaintext “abc...xyz”
- ▶ CCA: similar to CPA (the attack cannot be improved)
- ▶ similarly for shift cipher ...

Key length (symmetric schemes)

- ▶ generic attack: exhaustive search of key space (brute-force)
- ▶ large key space size: necessary but not sufficient requirement
- ▶ example of brute-force attack (key space covered):

time	key length [bits]
1 minute	33.7
1 hour	39.6
1 day	44.1
1 month	49.1
1 year	52.7

i7-2600, 4 cores, HT, HW accelerated AES, 225 mil. AES-128 decryptions/s

Data authenticity

- ▶ ensuring that received message is genuine (authentic)
- ▶ encryption alone does not guarantee authenticity
 - ▶ see one-time pad, ...
 - ▶ but: authenticated encryption
- ▶ MAC (message authentication code)
 - ▶ symmetric construction, MAC sent together with a message
 - ▶ verification: recomputing and comparing MAC
 - ▶ without non-repudiation
- ▶ digital signature scheme
 - ▶ asymmetric construction (public/private key)
 - ▶ anyone can verify a signature
 - ▶ only the owner of the private key can sign
- ▶ how to define security: goal and capabilities of an attacker?

Modern cryptology

1. formal security definitions
2. precise formulation of assumptions (attacker's capabilities, assumptions)
3. security proofs

How cryptography fails

- ▶ real-world security problems
- ▶ NIST's National Vulnerability Database (NVD)
- ▶ more than 25 000 vulnerabilities published in 2022
- ▶ usual problems related to cryptography:
 - ▶ bad randomness source for keys generation
 - ▶ insufficient checking of public-key certificates
 - ▶ incorrect implementation of cryptographic algorithms/protocols
 - ▶ fixed passwords of service accounts or passwords derived from public information

NVD – statistics for 2022 (selected CWEs)

Category	Count
CWE-311 Missing Encryption of Sensitive Data	37
CWE-326 Inadequate Encryption Strength	41
CWE-327 Use of a Broken or Risky Cryptographic Algorithm	58
CWE-338 Use of Cryptographically Weak Pseudo-Random Number Generator (PRNG)	14
CWE-347 Improper Verification of Cryptographic Signature	52
CWE-798 Use of Hard-coded Credentials	197
CWE-79 Improper Neutralization of Input During Web Page Generation ('Cross-site Scripting')	3293
CWE-20 Improper Input Validation	551
CWE-200 Exposure of Sensitive Information to an Unauthorized Actor	183
NVD-CWE-noinfo Insufficient Information	3255