Hash functions

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Content

Introduction

- hash function computes a fixed-length fingerprint/digest/hash from a message/document of (almost) arbitrary length
- $h : X \longrightarrow Y$ function (deterministic computation)
- efficient (fast) & no key used
- usually $X = \{0,1\}^*$, $X = \{0,1\}^{\le 2^{64}}$, $X = \{0,1\}^{\le 2^{128}}$, …
  $Y = \{0,1\}^{160}$ for SHA-1, $\{0,1\}^{256}$ for SHA-256 and SHA3-256, …
- various uses of h.f.:
  - digital signature schemes (digest of the message is signed)
  - padding in public-key encryption schemes
  - verifying integrity of data
  - instantiation of random oracles and pseudorandom functions
  - MAC constructions
  - password storing methods etc.

Basic requirements of hash functions (informally)

- preimage resistance (one-way)
  - It is infeasible to compute $x \in X$ given $y \in h(X)$ such that $h(x) = y$.
- second preimage resistance
  - It is infeasible to compute $x' \in X$ given $x \in X$ such that $x \neq x'$ & $h(x) = h(x')$.
- collision resistance
  - It is infeasible to compute $x, x' \in X$ such that $x \neq x'$ & $h(x) = h(x')$.

- remarks:
  - $|X| \gg |Y|$, otherwise the h.f. is useless $\Rightarrow$ large number of collisions
  - Y is finite, h is deterministic $\Rightarrow$ in theory, e.g. collisions can be found in $O(1)$ time ("hardcoded")
  - formalizing the requirements is not straightforward (introduction of *hash function families*, multiple "flavors" of preimage and second preimage resistance) – however, above intuition satisfies our needs
  - Pre, Sec, Coll, (aPre, ePre, aSec, eSec), MAC, Prf, Pro, TCR, CTFP, …

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Properties of h.f. – discussion

- collision resistance $\Rightarrow$ second preimage resistance
  - if you can find a second preimage, then you have a collision
- collision resistance $\not\Rightarrow$ preimage resistance
  - identity: $X = Y, \forall x \in X: \; h(x) = x$ (Coll, ¬Pre)
  - let g with range $\{0, 1\}^n$ be collision and preimage resistant; then

$$h(x) = \begin{cases} 0 \,||\, x & \text{if } |x| = n \\ 1 \,||\, g(x) & \text{otherwise} \end{cases}$$

    is collision resistant but not preimage resistant
- second preimage resistance $\not\Rightarrow$ preimage resistance
  - identity again (Sec, ¬Pre)
- however, in a "normal" situation …

Collision by inverting h.f.

- assumption: h can be inverted efficiently
- algorithm:
  1. $x \xleftarrow{\$} X$
  2. invert $h(x) \mapsto x'$
  3. if $x' \neq x$ …collision found
- let us estimate the probability of success
- notation: $[x] = \{x' \in X;\ h(x') = h(x)\}$ equivalence class
- C – set of all equivalence classes

$$\Pr_{\text{succ}} = \frac{1}{|X|}\sum_{x\in X}\frac{|[x]|-1}{|[x]|} = \frac{1}{|X|}\sum_{c\in C}\sum_{x\in c}\frac{|c|-1}{|c|} = \frac{1}{|X|}\sum_{c\in C}(|c|-1)$$

$$= \frac{1}{|X|}\Bigg(\underbrace{\sum_{c\in C}|c|}_{|X|} - \underbrace{\sum_{c\in C}1}_{\leq|Y|}\Bigg) \geq 1-\frac{|Y|}{|X|} \qquad \ldots \geq \underbrace{1-\left(\frac{|Y|}{|X|}\right)^k}_{\text{after } k \text{ repetitions}}$$

Generic attack for finding preimage/2nd preimage

- generic attack, finding a preimage for given $y \in h(X)$:
- algorithm:
  1. choose $x \in X$ (randomly or systematically)
  2. if $h(x) = y$ then the preimage is found, otherwise repeat
- expected complexity $O(2^n)$ for $Y = \{0, 1\}^n$
- similar generic attack for finding a second preimage

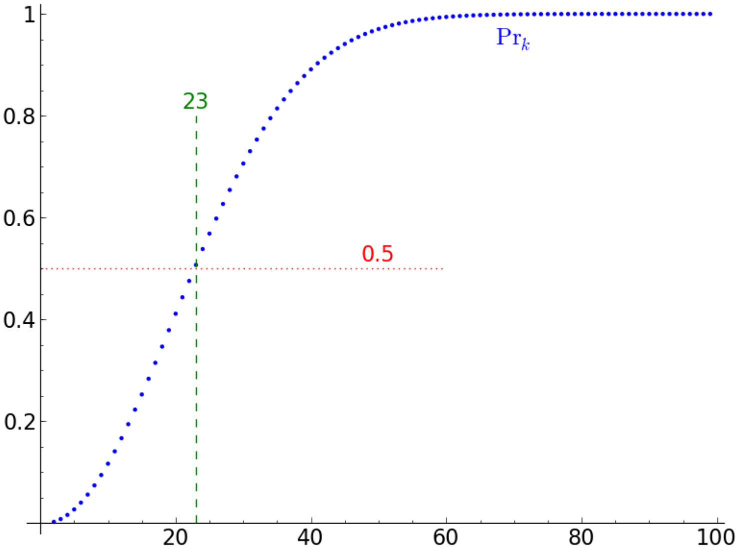
Birthday attack – introduction

- generic attack for finding collision(s)
- example: What is the probability that at least two people in a room share the same birthday?

$$\Pr_2 = 1 - \frac{365 \cdot 364}{365^2} \approx 0.0027; \quad \Pr_3 = 1 - \frac{365 \cdot 364 \cdot 363}{365^3} \approx 0.0082$$

- k people: $\Pr_k = 1 - 365^{\underline{k}}/365^k$
- at least 23 people needed for probability $\geq 1/2$
- "hash function" maps people to dates; $|Y| = 365$

Birthday attack – introduction (2)



Birthday attack on h.f.

1. choose (distinct) $x_1, \dots, x_k \xleftarrow{\$} X$
2. compute $h(x_1), \dots, h(x_k)$
3. find collisions, for example by sorting $(h(x_i), x_i)$ and searching for collisions in adjacent elements, or by storing $(h(x_i), x_i)$ in a hash table using the hash value as a key

- linear time and memory complexity $O(k)$
  - we treat n as a constant (for $Y = \{0,1\}^n$); also assuming constant time to evaluate h
  - time: using Radixsort for sorting in $O(k)$ or using a hash table with $k \times O(1)$ operations
  - memory complexity can be improved (see later)

What is the probability of success?

Birthday attack – analysis (1)

- trivial observations – the probability of success increases:
  - for increasing k
  - for unbalanced distribution of images
- assume the worst situation: h distributes the hash values uniformly, i.e.

$$\Pr[h(x) = y] = 1/|Y| \qquad \forall y \in Y$$

- let $y_1, \dots, y_k$ be random, independent and uniform elements from Y
- notation: $|Y| = N$
- probability that all y_i's are distinct:

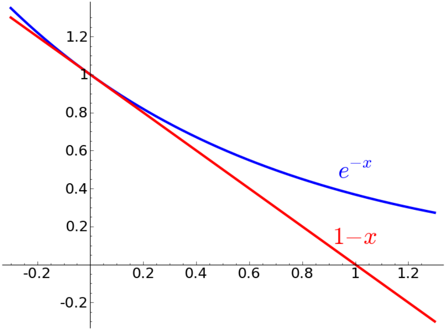
$$\Pr_{\text{dist}} = \frac{N(N-1)\dots(N-k+1)}{N^k} = \left(1 - \frac{1}{N}\right)\left(1 - \frac{2}{N}\right)\dots\left(1 - \frac{k-1}{N}\right)$$

Birthday attack – analysis (2)

- probability of at least one collision: $\Pr_{\text{col}} = 1 - \Pr_{\text{dist}}$
- estimate $\Pr_{\text{col}}$:

$$\Pr_{\text{col}} = 1 - \prod_{i=1}^{k-1}\left(1 - \frac{i}{N}\right) \geq 1 - e^{-\frac{1}{N}-\frac{2}{N}-\dots-\frac{k-1}{N}} = 1 - e^{\frac{-k(k-1)}{2N}}$$

we use inequality $1 - x \leq e^{-x}$
it follows from Taylor series:
$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$



Birthday attack – analysis (3)

- find k such that $\Pr_{\text{col}} \geq \varepsilon$, for some constant $\varepsilon \in (0, 1)$

$$
\begin{aligned}
\Pr_{\text{col}} \geq 1 - e^{-k(k-1)/(2N)} &\geq \varepsilon \\
1 - \varepsilon &\geq e^{-k(k-1)/(2N)} \\
2N \ln(1 - \varepsilon) &\geq -k^2 + k \\
k^2 - k + 2N \ln(1 - \varepsilon) &\geq 0 \\
k &\geq \frac{1}{2} + \sqrt{\frac{1}{4} + 2N \ln \frac{1}{1 - \varepsilon}} \\
k &\geq \sqrt{N} \cdot \sqrt{2 \ln \frac{1}{1 - \varepsilon}}
\end{aligned}
$$

Birthday attack – results

- the complexity of b.a. for “reasonable” ε, e.g. $1/2$, $2/3$, is $O(N^{1/2})$
- for $Y = \{0, 1\}^n$ we get $\approx 2^{n/2}$ (e.g. for SHA-1 $\approx 2^{80}$)
- expected k for given success probability:

$$
\begin{array}{ll}
50\% & k \approx 1.177 \cdot 2^{n/2} \\
90\% & k \approx 2.146 \cdot 2^{n/2} \\
99\% & k \approx 3.035 \cdot 2^{n/2}
\end{array}
$$

Implications of birthday attack

- generic attack, i.e. any h.f. can be attacked this way
  - recall: generic attack for symmetric encryption is brute-force, $O(2^k)$ for key length k
- the length of hash value (digest) should be twice the length of symmetric key used for encryption
- standardized parameters of AES and SHA-2 family:

AES key length		128	192	256
SHA-2 output length	$224^{(*)}$	256	384	512

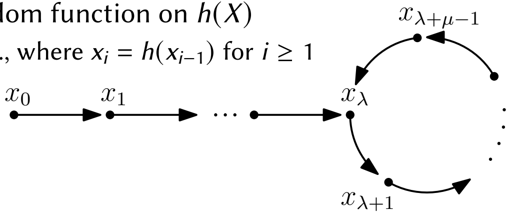
$^{(*)}$ this corresponds to the effective key length of 3DES (112 bits)

“Meaningful” collisions

- prepare documents m, m' with t places that can be changed without changing the meaning of the document
  - one space vs. two spaces, synonyms etc.
- 2^t variants of each document
- hash and find a collision between these two sets
- the same asymptotic time and memory complexity of b.a.

Improving memory complexity of birthday attack (1)

- assumption: h as a random function on $h(X)$
  - sequence: $x_0, x_1, x_2 \ldots$, where $x_i = h(x_{i-1})$ for $i \geq 1$



  - expected (as $N \to \infty$): $\rho = \lambda + \mu = \sqrt{\pi N/2}$
- finding collision in constant memory:
  1. $x_0 \xleftarrow{\$} X$ (using $X \setminus Y$ guarantees the existence of a collision, $\lambda \geq 1$)
  2. compute (x_i, x_{2i}) for $i \geq 1$: $x_i = h(x_{i-1})$, $x_{2i} = h(h(x_{2(i-1)}))$
  3. if $x_i = x_{2i}$ then $h^i(x_0) = h^{2i}(x_0)$, we found a point on the cycle, $\lambda \leq i$, and the collision can be computed as follows:
     - 3.1 compute (x_j, x_{i+j}) for $j = 0, 1, \ldots, i$ starting with (x_0, x_i)
     - 3.2 check for situation when $x_j \neq x_{i+j}$ and $x_{j+1} = x_{i+j+1}$
     - 3.3 collision $h(x_j) = h(x_{i+j})$; remark: $\mu \mid (2i - i) \implies x_\lambda = x_{i+\lambda}$

Improving memory complexity of birthday attack (2)

- only a constant number of values (e.g. x_0, and the recent pair of values (x_i, x_{2i}) or (x_j, x_{i+j})) should be stored
- complexity:
  - cycle is detected (point is found) if $i \geq \lambda$ and $\mu \mid i$
  - the difference $2i - i$ increases by 1 in each iteration, i.e. the cycle is detected with $\lambda + \mu$ iterations maximum
  - complexity $O(\lambda + \mu) = O(\sqrt{N})$
- this method does not change the asymptotic time complexity of b.a.
- no control over the colliding messages/inputs

Collision resistance in practice

- collision resistance is not easy
- MD5
  - designed by Ron Rivest (1991)
  - collision published in 2005
- SHA-1
  - designed by NSA, published as a standard in 1995
  - deprecated in major web browsers in 2017
  - first collision published in 2017; two pdf files, see https://shattered.io/
  - attack complexity: $2^{63.1}$ SHA-1 compressions
- hash functions in web servers certificates (for signatures):

	01/2015	01/2016	01/2017	01/2018	01/2019
SHA-1	66.7%	13.2%	1.5%	0.0%	0.0%
SHA-256	33.3%	86.8%	98.4%	99.8%	99.8%

source: SSL Pulse, https://www.ssllabs.com/ssl-pulse/

Hash functions based on hard problems

- provable properties (assuming the hardness of underlying problem)
- slow, impractical $\Rightarrow$ not used in practice
- example based on discrete logarithm problem:
  - $(G, \cdot)$ – group of prime order p; let g be a generator of $(G, \cdot)$
  - $f \in G$, such that $\alpha = \log_g f$ is unknown
  - $h : \mathbb{Z}_p \times \mathbb{Z}_p \longrightarrow G$ is defined as follows: $h(a, b) = g^a \cdot f^b$
  - h is collision resistant, otherwise we can find α:

$$
\begin{aligned}
h(a, b) &= h(a', b') \qquad \text{where } (a, b) \neq (a', b') \\
g^a \cdot f^b &= g^{a'} \cdot f^{b'} \\
g^{a+\alpha b} &= g^{a'+\alpha b'} \quad \Rightarrow \quad \alpha = \frac{a - a'}{b' - b} \mod p
\end{aligned}
$$

Hash functions based on block ciphers

- $m = m_1, m_2, \ldots, m_k$ input divided into blocks
- h_0 – initialization vector; h_i – intermediate hash value ($1 \leq i \leq k$)
- iteration – sequential processing of input blocks
- examples:
  - Matyas, Meyer, Oseas: $h_i = E_{g(h_{i-1})}(m_i) \oplus m_i$
  - Davies, Meyer: $h_i = E_{m_i}(h_{i-1}) \oplus h_{i-1}$
  - Miyaguchi, Preneel: $h_i = E_{g(h_{i-1})}(m_i) \oplus h_{i-1} \oplus m_i$
- $H(m) = h_k$ (the hash value is the output of the last iteration)
- problem: standard block ciphers have small block length
  - specific block ciphers (SHACAL for SHA-1, W cipher for Whirlpool etc.)
  - double block length constructions (MDC-4, Hirose, Tandem-DM etc.)

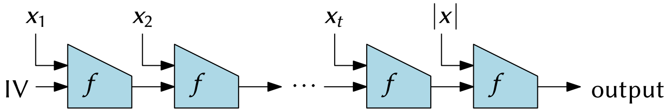
Dedicated constructions

- no proofs of security based on some “hard underlying problem”
- fast (usually one of the design goals)
- iterated construction (informally):
  - message padding and “slicing”
  - start with IV and sequentially process the slices
  - result is the output of the final iteration (sometimes after some additional processing)
- most common approaches
  - Merkle-Damgård: SHA-1, SHA-2 family
  - sponge: SHA-3 (Keccak)

Merkle-Damgård construction (1)

- collision resistance of compression function implies collision resistance of hash function
- fixed input length compression function $f : \{0,1\}^{n+r} \longrightarrow \{0,1\}^n$
- hash function $H : \{0,1\}^{\leq l} \longrightarrow \{0,1\}^n$
- input $x = x_1, x_2, \dots, x_t$ (block length r)
  - last block padded by $10\dots0$ (if needed)
  - additional block $x_{t+1} = |x|$; in binary, thus $l < 2^r$
- other variants of padding used in practice or proposed in the literature
- using the length in padding $\sim$ MD strengthening
  - ensures suffix-free property of the padding:
    for any $x \neq x'$, $\mathrm{pad}(x)$ is not a suffix of $\mathrm{pad}(x')$
  - suffix-free $\sim$ necessary and sufficient condition for collision-preserving padding

Merkle-Damgård construction (2)



computation:

1. $h_0 = 0^n$ (initialization vector)
2. $h_i = f(h_{i-1} \,||\, x_i)$, for $i = 1, \ldots, t+1$
3. $H(x) = h_{t+1}$

Collision resistance of MD construction

let $x \neq x'$ be a collision in H: $H(x) = H(x')$, i.e. $h_{t+1} = h'_{t'+1}$

a. if $t \neq t'$ then $x_{t+1} \neq x'_{t'+1}$ and $f(h_t, x_{t+1}) = f(h'_{t'}, x'_{t'+1})$ …collision in f

b. $t = t'$: $x = x_1, \ldots, x_{t+1}$, $x' = x'_1, \ldots, x'_{t+1}$
   $f(h_t, x_{t+1}) = f(h'_t, x'_{t+1})$ …either collision in f or
   - $h_t = h'_t$ & $x_{t+1} = x'_{t+1}$
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- either we get a collision in f or $x = x'$

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Parameters of real-world hash function

family	function	length [bits]		
		max. input	output	block
	MD-5	$2^{64}-1$	128	512
	SHA-1	$2^{64}-1$	160	512
	Whirlpool	$2^{256}-1$	512	512
SHA-2	SHA-256	$2^{64}-1$	256	512
	SHA-384	$2^{128}-1$	384	1024
	SHA-512	$2^{128}-1$	512	1024
SHA-3	SHA3-256	∞	256	1088
	SHA3-384	∞	384	832
	SHA3-512	∞	512	576

SHA-2

- SHA-2 family of hash function
  (SHA-224, SHA-256, SHA-384, SHA-512, SHA-512/224 and SHA-512/256)
- standard: FIPS PUB 180-4
- similar design of SHA-256 (32-bit words, block size 512 bits) and SHA-512 (64-bit words, block size 1024 bits)
- other variants are truncated versions with different initialization vectors
- Merkle-Damgård construction

Example: SHA-256

- input message M; $l = |M|$ $(0 \leq l < 2^{64}$ bits)
- padding and parsing:
  - padding: $M1\underbrace{00\ldots0}_{k}\underbrace{(l)_2}_{64\text{ bits}}$, where k is the smallest value such that the overall length is a multiple of 512
  - parsing into 512-bit blocks: $M^{(1)}, M^{(2)}, \ldots, M^{(N)}$
  - each block consists of 16 32-bit words: $M^{(i)} = M_0^{(i)}, M_1^{(i)}, \ldots, M_{15}^{(i)}$
- initialization vector (8 32-bit words): $H_0^{(0)}, H_1^{(0)}, \ldots, H_7^{(0)}$
- intermediate hash values: $H_0^{(i)}, H_1^{(i)}, \ldots, H_7^{(i)}$
- SHA-256 digest: $H_0^{(N)}, H_1^{(N)}, \ldots, H_7^{(N)}$

SHA-256 compression function

compression function (for $i = 1, \dots, N$):

1. expanding a message block ($\mapsto W_0, \dots, W_{63}$)

$$W_i = \begin{cases} M_t^{(i)} & \text{for } 0 \le t \le 15 \\ \sigma_1(W_{t-2}) + W_{t-7} + \sigma_0(W_{t-15}) + W_{t-16} & \text{for } 16 \le t \le 63 \end{cases}$$

2. $(a, b, c, d, e, f, g, h) \leftarrow (H_0^{(i-1)}, H_1^{(i-1)}, \dots, H_7^{(i-1)})$
3. for $t = 0, \dots, 63$:
   3.1 $T_1 = h + \sum_1(e) + \mathrm{Ch}(e, f, g) + K_t + W_t$
   3.2 $T_2 = \sum_0(a) + \mathrm{Maj}(a, b, c)$
   3.3 $(a, b, c, d, e, f, g, h) \leftarrow (T_1 + T_2, a, b, c, d + T_1, e, f, g)$
4. $(H_0^{(i)}, H_1^{(i)}, \dots, H_7^{(i)}) \leftarrow (a + H_0^{(i-1)}, b + H_1^{(i-1)}, \dots, h + H_7^{(i-1)})$

SHACAL-2 block cipher in Davies-Meyer mode

Functions used in SHA-256

- functions operate on 32-bit words
- addition is computed mod 2^{32}
- $\mathrm{Ch}(x, y, z) = (x \wedge y) \oplus (\neg x \wedge z)$
- $\mathrm{Maj}(x, y, z) = (x \wedge y) \oplus (x \wedge z) \oplus (y \wedge z)$
- $\sum_0(x) = \mathrm{ROTR}^2(x) \oplus \mathrm{ROTR}^{13}(x) \oplus \mathrm{ROTR}^{22}(x)$
- $\sum_1(x) = \mathrm{ROTR}^6(x) \oplus \mathrm{ROTR}^{11}(x) \oplus \mathrm{ROTR}^{25}(x)$
- $\sigma_0(x) = \mathrm{ROTR}^7(x) \oplus \mathrm{ROTR}^{18}(x) \oplus \mathrm{SHR}^3(x)$
- $\sigma_1(x) = \mathrm{ROTR}^{17}(x) \oplus \mathrm{ROTR}^{19}(x) \oplus \mathrm{SHR}^{10}(x)$
- ROTR – circular shift rotation to the right
- SHR – shift to the right

Some performance numbers

	MB/s
MD5	687
SHA-1	738
SHA-256	323
SHA-512	417
SHA3-256	287
SHA2-512	154

block size: 8192 bytes, 1 thread
platform: i7-2600 @ 3.40 GHz (4 cores/8 threads, AES-NI)
implementation: openssl 1.1.1c

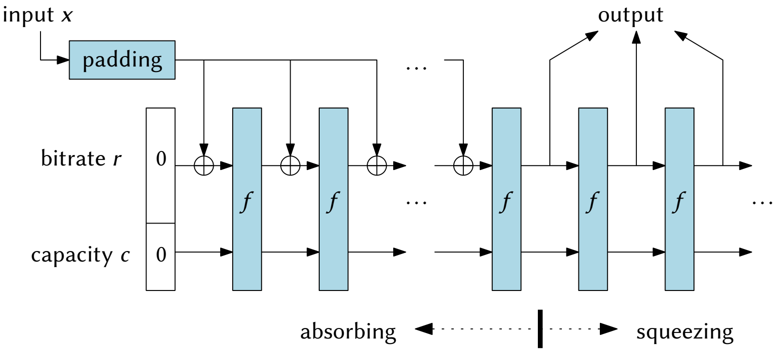
Remark: Intel SHA Extensions – instructions for improving performance of SHA-1 and SHA-256 hash functions (not used in above table); AMD Ryzen and few Intel processors.

SHA-3 overview

- Keccak – winner of SHA-3 competition (2012)
- standard: NIST FIPS 202 (2015)
  - 4 hash functions with fixed-length output: SHA3-224, SHA3-256, SHA3-384, SHA3-512
  - 2 functions with variable-length output (XOF – extendable-output functions): SHAKE128, SHAKE256
- different approach than SHA-1 or SHA-2 hash functions
- Keccak is not an MD-construction
- sponge construction
- other functions/variants/constructions proposed:
  - SHA-3 Derived Functions: cSHAKE, KMAC, TupleHash and ParallelHash (NIST SP 800-185, 2016)

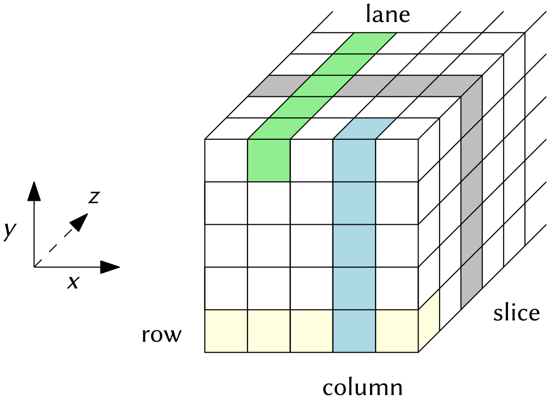
SHA-3 structure

- sponge construction – absorbing & squeezing
  - arbitrary output length
  - f – permutation on $\{0,1\}^{r+c}$
  - r – bitrate (e.g. 1088 for SHA3-256)
  - c – capacity (e.g. 512 for SHA3-256)
  - padding for SHA3-256: $x \,||\, 01 \,||\, 10^{*}1$



SHA-3 inside permutation f (1)

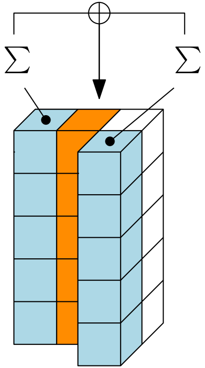
- state: $5 \times 5 \times 2^l$ bits ($2^l = 64$ for SHA3-256)



- $12 + 2l$ rounds (24 rounds for SHA3-256)
- round function: $R = \iota \circ \chi \circ \pi \circ \rho \circ \theta$, ($\theta$ is applied first)

SHA-3 inside permutation f (2)

- θ (theta) – xor each bit of a column with parities of two neighboring columns
- ρ (rho) – rotate each lane by a constant value
- π (pi) – permute the positions of the lanes
- χ (chi) – flip bit if neighbors to the right are 0, 1
  - χ operates on rows (independently, in parallel)
- ι (iota) – xor a round specific constant to lane[0,0]
  - destroying symmetry



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