Block Ciphers I

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Content

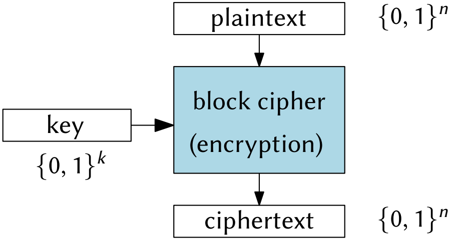
Introduction
- iterated ciphers, Feistel ciphers
- Simon and Speck

AES (Advanced Encryption Standard)
- description
- security

Multiple encryption
- meet in the middle attacks, 3DES

Slide attack

Introduction - Block ciphers



- encryption/decryption $E, D : \{0,1\}^k \times \{0,1\}^n \longrightarrow \{0,1\}^n$
- k – key length, n – block length
- correctness: $\forall K \in \{0,1\}^k \ \forall m \in \{0,1\}^n : \ D_K(E_K(m)) = m$
- E_K and D_K are mutually inverse permutations on $\{0,1\}^n$

Block ciphers – examples

- more versatile than stream ciphers (modes of operation)
- used more often than stream ciphers
- AES – block length: 128, key lengths: 128, 192, 256
- 3DES (also TDEA) – block length: 64, key lengths: 112, 168
- NIST SP 800-131A rev. 2 (2019):
  - AES acceptable
  - 3DES (with 168-bits keys) deprecated through 2023, disallowed after 2023
- ISO standardized the following block ciphers:
  - ISO/IEC 18033-3:2010
    64-bits block: TDEA, MISTY1, CAST-128, HIGHT
    128-bits block: AES, Camellia, SEED
  - ISO/IEC 29192-2:2019 (Lightweight cryptography)
    64-bits block: PRESENT
    128-bits block: CLEFIA, LEA
- standardized $\not\Longrightarrow$ used

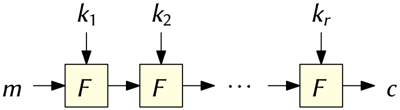
Block ciphers – remarks

- alternative view: block cipher as a simple substitution
  - huge alphabet, frequency analysis impossible
- short block size – (possibly) easier cryptanalysis
- extremely short block size
  - small alphabet
  - max. $(2^n)!$ permutations, regardless of key length
- extremely short key length:
  - exhaustive key search (brute-force attack) $\sim 2^k$

Security

- exhaustive key search (EKS) complexity $\sim 2^k$
- expected EKS complexity $\sim 2^{k-1}$
- important assumption: keys with uniform distribution (!)
  - otherwise enumerate keys by their probabilities (in descending order)
  - keys often derived from user passwords ($\Rightarrow$ non-uniformity)
- (almost) anything with better complexity than EKS is a successful cryptanalytic attack (at least in theory)
- can be still impractical, because of
  - complexity, e.g. 2^{120} instead of 2^{128} is still infeasible
  - assumptions, e.g. CPA with 2^{90} of chosen plaintext blocks encrypted with the same key is rather unrealistic

Iterated ciphers



- the most frequently used construction method for block ciphers
- iteration of round function $F : \{0,1\}^{k'} \times \{0,1\}^n \to \{0,1\}^n$
- structure:
  - key scheduling/expansion: producing round keys $k_1, \dots, k_r$ from the key
  - sequential iteration of F (r rounds): $c = F_{k_r}(\dots F_{k_2}(F_{k_1}(m)) \dots)$
  - usually with some form of key whitening: $c = k_{r+1} \oplus F_{k_r}(\dots F_{k_1}(m \oplus k_0) \dots)$
  - sometimes the first/the last round is different
- decryption employs inverse round function
- examples: AES-128 has 10 rounds, PRESENT has 32 rounds

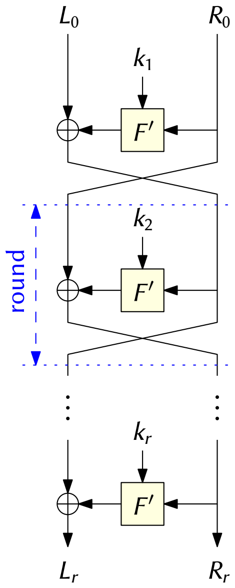
Feistel ciphers

- method of constructing a round function (its inverse has the same structure)
- decryption ∼ encryption (with reversed order of round keys) ⇒ equal speed of encryption and decryption with pre-computed round keys
- plaintext divided into left and right halves: L_0, R_0
- iterations (for $i = 1, \dots, r-1$):

$$\begin{aligned} L_i &= R_{i-1} \\ R_i &= L_{i-1} \oplus F'_{k_i}(R_{i-1}) \end{aligned}$$

- last round:

$$\begin{aligned} L_r &= L_{r-1} \oplus F'_{k_r}(R_{r-1}) \\ R_r &= R_{r-1} \end{aligned}$$



Feistel ciphers – decryption

- using the same structure, changing the order of round keys
- denote $L'_0 = L_r$ and $R'_0 = R_r$ (and other intermediate values L'_i, R'_i)
- we can show that $L'_i = R_{r-i}$ and $R'_i = L_{r-i}$ for $i = 1, \dots, r-1$:
  - the first round:

$$L'_1 = R'_0 = R_r = R_{r-1}$$
$$R'_1 = L'_0 \oplus F'_{k_r}(R'_0) = L_r \oplus F'_{k_r}(R_{r-1}) = L_{r-1}$$

  - the second round (other rounds similarly):

$$L'_2 = R'_1 = L_{r-1} = R_{r-2}$$
$$R'_2 = L'_1 \oplus F'_{k_{r-1}}(R'_1) = R_{r-1} \oplus F'_{k_{r-1}}(R_{r-2}) = L_{r-2}$$

- the last rounds (assuming $L'_{r-1} = R_1$ and $R'_{r-1} = L_1$):

$$R'_r = R'_{r-1} = L_1 = R_0$$
$$L'_r = L'_{r-1} \oplus F'_{k_1}(R'_{r-1}) = R_1 \oplus F'_{k_1}(L_1) = L_0$$

Feistel ciphers – remarks

- examples: DES (3DES), Camellia, Blowfish, etc.
- generalizations:
  unbalanced Feistel (splitting block into parts of unequal length)
- Feistel network is used in other cryptographic constructions, e.g.:
  - OAEP (Optimal Asymmetric Encryption Padding) for RSA encryption
  - format preserving encryption
- theoretical construction:

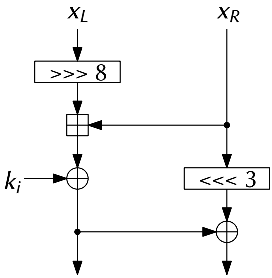
  pseudorandom function $\rightarrow$ pseudorandom permutation

Simon and Speck

- lightweight block ciphers
  - families, variants with various block and key sizes
  - both ciphers with excellent performance in HW and SW
- published by NSA (2013)
- Simon
  - optimized for hardware, balanced Feistel network
  - XOR, bitwise AND, ROT (rotation)
- Speck
  - optimized for software, ARX cipher (modular addition, XOR, ROT)
- proposed as ISO standard in 2014
  - rejected in 2018 by subcommittee ISO/IEC JTC 1/SC 27 (Information security, cybersecurity and privacy protection)
  - standardized later in 2018 by other subcommittee ISO/IEC JTC 1/SC 31 (Automatic identification and data capture techniques)

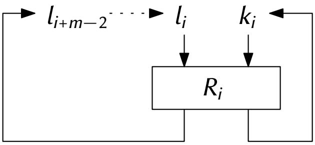
Speck

- 10 variants of block/key lengths
  - starting with 32-bit block and 64-bit key …
  - e.g. 128-bit block with 128, 192, or 256-bit key (32, 33, 34 rounds)
- Speck2n
  - 2n-bit block (two n-bit words)
  - round function (round key k_i):

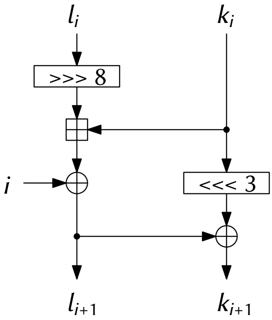


Speck – key expansion

- a key K consists of m words, $m \in \{2, 3, 4\}$ ($m = |K|/n$)
  - for example: $m = 2$ for Speck128/128, $m = 4$ for Speck128/256
- $K = (l_{m-2}, \ldots, l_0, k_0)$
- round function is used for key expansion:



general scheme



case $m = 2$

AES (Advanced Encryption Standard)

- DES deficiency: short key length (56 bits)
- public standardization process for new encryption standard (1997–2000)
- requirements: block cipher, block length 128 bits, key lengths 128, 192, 256 bits
- Rijndael – winning algorithm (Vincent Rijmen, Joan Daemen)
- NIST standardized AES in 2001 (other standardizations followed)
- the most important symmetric cipher today
- used (almost) everywhere

AES

- **not** a Feistel cipher
- different number of rounds depending on key length:
  AES-128 10 rounds, AES-192 12 rounds, AES-256 14 rounds
- slight performance degradation for longer key lengths

	1 thread (millions AES/s)	
	with AES-NI	no AES-NI
AES-128	42.8	7.1
AES-192	36.1	6.0
AES-256	31.4	5.3

platform: i7-2600 @ 3.40 GHz (4 cores/8 threads, AES-NI)
implementation: openssl 1.0.1
overall encryption performance AES-128: 0.75 GB/s (1 thread, AES-NI)

AES – state and operations

- state (plaintext, internal state, ciphertext): 4×4 array of bytes:

0 $s_{0,0}$	4 $s_{0,1}$	8 $s_{0,2}$	12 $s_{0,3}$
1 $s_{1,0}$	5 $s_{1,1}$	9 $s_{1,2}$	13 $s_{1,3}$
2 $s_{2,0}$	6 $s_{2,1}$	10 $s_{2,2}$	14 $s_{2,3}$
3 $s_{3,0}$	7 $s_{3,1}$	11 $s_{3,2}$	15 $s_{3,3}$

row

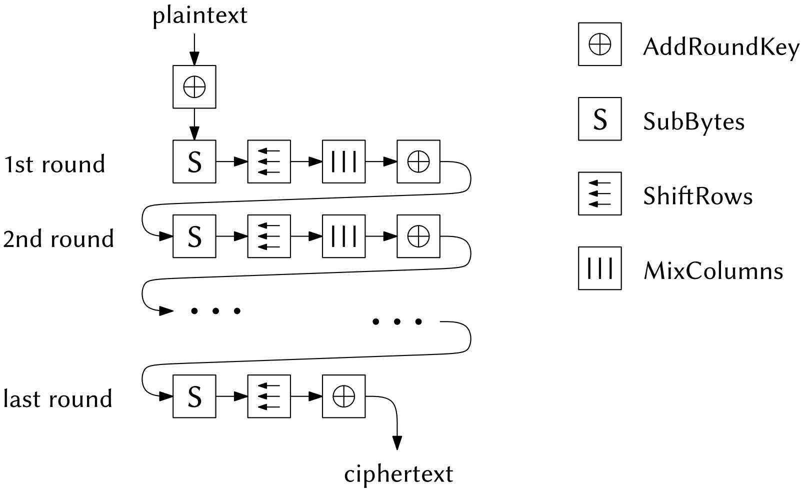
column

- 4 basic operations (invertible):
  1. AddRoundKey – XOR the state with 128-bit round key
  2. SubBytes – replace each byte using a fixed permutation (S-box)
  3. ShiftRows – cyclically shift each row of the state
  4. MixColumns – multiply each column by a fixed matrix

AES – details of operations

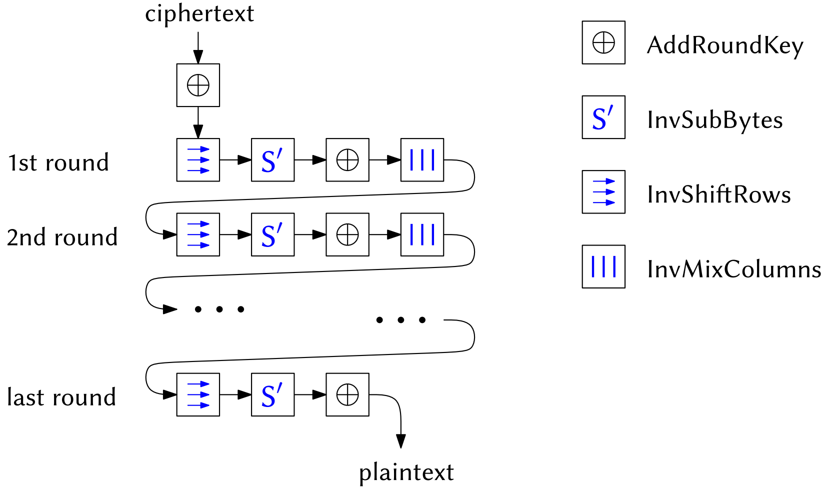
1. AddRoundKey: fast mix of round key in; self-inverse
2. SubBytes: $s_{i,j} = S(s_{i,j})$ for all $0 \leq i, j \leq 3$
   - the only nonlinear operation in AES
   - carefully chosen (a linear/affine ciphers are easy to break)
   - invertible: inverse permutation on $\{0, 1\}^8$
3. ShiftRows:
   - 1st row is not shifted
   - 2nd/3rd/4th row: bytes are cyclically shifted to the left by 1/2/3 bytes
   - example: $(s_{1,0}, s_{1,1}, s_{1,2}, s_{1,3}) \mapsto (s_{1,1}, s_{1,2}, s_{1,3}, s_{1,0})$
   - invertible: shift to the right
4. MixColumns
   - fixed (invertible !) matrix M
   - good diffusion properties (small difference on input “amplifies”)

AES – encryption structure



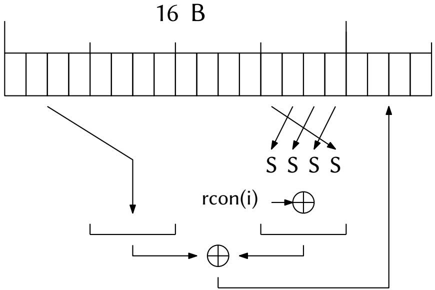
AES – decryption structure

inverse operations: InvShiftRows, InvMixColumns, InvSubBytes



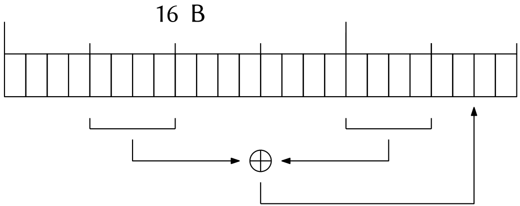
AES – key expansion (for 128 bit key) 1

- AES-128 $\Rightarrow$ 10 rounds $\Rightarrow$ 11 round keys (i.e. $11 \cdot 16 = 176$ B)
- first 16 B (first round key) is the encryption key
- rcon(i) – round constant
- 1st 4-byte word in each new round key:



AES – key expansion (for 128 bit key) 2

- for the 2nd, 3rd and 4th 4-byte word in each round key:



- round keys are formed from consecutive bytes of the expanded key
- slightly different key expansion for key length 256

AES – security

- exhaustive key search complexity $\sim 2^{128}$ or 2^{192} or 2^{256}
- best key recovery attacks
  - Bogdanov et al. 2011, KPA:

	time	data
AES-128	$2^{126.2}$	2^{88}
AES-192	$2^{189.7}$	2^{80}
AES-256	$2^{254.4}$	2^{40}

  - Tao and Wu 2015, KPA:

	time	data
AES-128	$2^{126.1}$	2^{56}
AES-192	$2^{189.9}$	2^{48}
AES-256	$2^{254.3}$	2^{40}

Multiple encryption

- multiple encryption (cascade encryption)
  - using the same or different ciphers, usually with independent keys

$$E_{k_1,k_2}(p) = E'_{k_2}(E^*_{k_1}(p))$$

- possible goals:
  - increasing the key space
  - security (what if a cipher is broken ... use two or three distinct)
- some ciphers cannot be strengthened (the key space does not increase), regardless of cascade length
  - examples: simple substitution, Vigenere, permutation, Vernam, etc.

$$\forall k_1, k_2 \; \exists k \; \forall p : E_{k_1}(E_{k_1}(p)) = E_k(p)$$

- independence of keys can be crucial
  - example: using the same key in double Vernam cipher $\Rightarrow$ no encryption

3DES (TDEA)

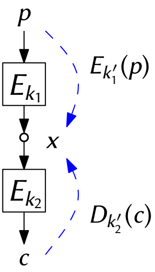
- 3DES is defined as a cascade of the length 3:
  - encryption: $E_{k_3}(D_{k_2}(E_{k_1}(p)))$
  - decryption: $D_{k_1}(E_{k_2}(D_{k_3}(c)))$
- keying options (and corresponding key length):
  - option 1: independent keys (168 bits)
  - option 2: $k_1 = k_3$ (112 bits)
  - option 3: $k_1 = k_2 = k_3$ (56 bits)
- EDE mode (instead of EEE mode) and keying option 3 ensures backward compatibility with DES
- real strength (bit security) of 3DES:
  - option 1: 112 bits (meet in the middle attack)
  - option 2: 80 bits (assuming 2^{40} known plaintext/ciphertext pairs)

Meet in the middle attack (MITM)

- disadvantage of multiple encryption – slower than single encryption
- why not “double encryption” – MITM attack
  - MITM is generally applicable to multiple encryption schemes
  - MITM is known plaintext attack (several pairs (p_i, c_i) known)

$c = E_{k_2}(E_{k_1}(p))$

1. $\forall k_2'$: compute $x = D_{k_2'}(c)$ and store (x, k_2') in a hash table indexed by x
2. $\forall k_1'$: compute $x = E_{k_1'}(p)$
   2.1 find entry(ies) (x, k_2') in the table
   2.2 verify a candidate key(s) (k_1', k_2') using other plaintext/ciphertext pairs



MITM – complexity

- assume key length k and block length n
- expected number of required plaintext/ciphertext pairs $\lceil 2k/n \rceil$
  - $\approx 2^{2k}/2^n$ “valid” key pairs for a single (p, c) pair
  - $\approx 2^{2k}/2^{tn}$ for t plaintext/ciphertext pairs
  - from $1 \sim 2^{2k}/2^{tn}$ we get $t \sim 2k/n$
- time complexity $O(2^k)$
  - first cycle 2^k iterations; second cycle 2^k iterations
  - single hash table operation $O(1)$
- space complexity $O(2^k)$
  - each key k_2' produces one fixed-length entry in the hash table
  - second cycle in constant memory
- easily generalized for longer cascades
  - example: MITM on 3DES with 3 keys – time 2^{112} and space 2^{56}

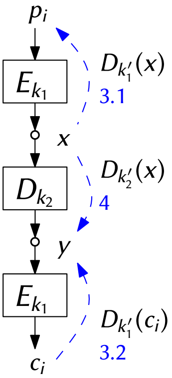
A KPA on two-key triple encryption

- assume t plaintext/ciphertext pairs $(p_i, c_i)_{i=1}^t$
- 3DES with keying option 2: $c = E_{k_1}(D_{k_2}(E_{k_1}(p)))$

1. build a hash table T1 with $(p_i, c_i)_{i=1}^t$ indexed by plaintexts

repeat steps 2–4 until the key pair is recovered:

2. choose random x
3. $\forall k_1'$:
   3.1 compute $p' = D_{k_1'}(x)$ and find $(p' = p_i, c_i) \in$ T1 (if exists)
   3.2 compute $y = D_{k_1'}(c_i)$, put (y, k_1') into hash table T2 indexed by y
4. $\forall k_2'$: compute $D_{k_2'}(x)$, find entry in T2, verify (k_1', k_2') pair



Analysis of the attack

- time complexity:
  - building T1: $O(t)$
  - for single x: step 3 $O(2^k)$, step 4 $O(2^k)$
  - attack succeeds iff x is an intermediate value for some (p_i, c_i)
  - expected number of tries to "guess" correct x (non-repeated guesses):
    - $(2^n - t)$ random variables X_z (for incorrect "middle" values)
    - X_z is 0/1 r.v. (0 iff z is not guessed before any correct x)
    - Ex denotes an expected value of random variable, $\mathrm{Ex}(X_z)$ is constant for all z
    - $\mathrm{Ex}(X_z) = 1/(t+1)$ (counting arrangements of z and t correct values)
    - guesses: $1 + \mathrm{Ex}(\sum_z X_z) = 1 + (2^n - t) \cdot \mathrm{Ex}(X_z) \approx (2^n - t)/(t+1) \approx 2^{n - \lg t}$
  - let's summarize: $O(t + 2^k \cdot 2^{n-\lg t}) = O(t + 2^{k+n-\lg t})$
- space complexity: $O(t + 2^{k-n} \cdot t)$
  - T1: $O(t)$ fixed-length entries;
  - T2 (new one for each x): expected number of entries $\approx (2^k/2^n) \cdot t$
  - space for T2 can be reused (not needed after finishing with x)
- easily adjustable for EEE (instead of EDE) mode of triple encryption

Examples

- 3DES with two key option:
  - parameters: $k = 56$, $n = 64$, $t = 2^{40}$
  - time complexity: $O(t + 2^{k+n-\lg t}) \approx 2^{120-40} = 2^{80}$
  - space complexity: $O(t + 2^{k-n} \cdot t) \approx 2^{40}$
- Triple AES-128 (not used in practice) with two-key option:
  - parameters: $k = 128$, $n = 128$, $t = 2^{60}$
  - time complexity: $O(t + 2^{k+n-\lg t}) \approx 2^{256-60} = 2^{196}$
  - space complexity: $O(t + 2^{k-n} \cdot t) \approx 2^{60}$
- different trade-offs for different t values

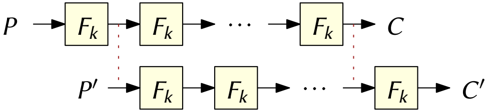
Data requirements of KPA/CPA

- assumption: block length $n = 128$
- only the ciphertext is considered for size computation, and for calculation of transmission time

data	size [TB]	time for 1Gb/s
2^{40}	17.6	39 hours
2^{60}	$1.8 \cdot 10^{8}$	4676 years
2^{80}	$1.9 \cdot 10^{13}$	$4.9 \cdot 10^{9}$ years
2^{100}	$2.0 \cdot 10^{19}$	$5.1 \cdot 10^{15}$ years

Slide attack 1

- iterated ciphers
  - easy to change the number of rounds
  - usually more rounds $\sim$ increased security
- Biryukov, Wagner (1999)
  - general attack on iterated cipher with identical round transform
  - arbitrary number of rounds
  - other variants exist
- cipher: $C = F_k \circ F_k \circ \ldots \circ F_k(P)$
- *slid pair* is a known pair of (P, C) and (P', C') such that $P' = F_k(P)$ and $C' = F_k(C)$



Slide attack 2

- we assume that F_k is “weak”:
  - easy to compute k from equations $y_0 = F_k(x_0)$, $y_1 = F_k(x_1)$
  - usually very easy; for example, try this for Speck2n or AES
- KPA attack
  - approx. $2^{n/2}$ of known plaintext-ciphertext pairs
    expecting ≈ 1 slid pair (birthday paradox)
  - testing all combinations if there is a slid pair (P, C), (P', C')
    Is there k such that $P' = F_k(P) \;\wedge\; C' = F_k(C)$? …($\approx 2^n$)
  - one slid pair recovers approx. n bits of the key
- Why bother when time complexity is $O(2^n)$?
  - single round (slide attack) vs. full cipher (brute-force)
  - other improvements depending on F

Slide attack 3

- KPA and CPA slide attacks much better with Feistel ciphers
  - single round …half of the block does not change
  - $\approx 2^{n/4}$ plaintext-ciphertext pairs for finding a slid pair
  - i.e. complexity is $O(2^{n/2})$
- advanced variants of slide attack exist
- pay attention to key scheduling