Stream Ciphers

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- RC4
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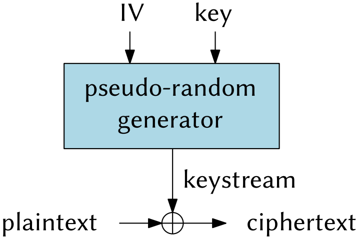
Introduction

- Vernam cipher (one-time pad)
  - perfect secrecy
  - impractical – long key that cannot be reused
- (some) stream ciphers examples:
  - RC4 – old software and protocols, e.g. WEP, SSL/TLS etc.
  - A5/1 – GSM communication (phone $\leftrightarrow$ base station)
    remark: UMTS and LTE use other ciphers
  - E0 – Bluetooth (BR/EDR – basic rate/enhanced data rate)
    remark: Bluetooth Low Energy uses AES-CCM
  - ChaCha20 – TLS (RFC 7905)
- basic types of stream ciphers: synchronous and self-synchronizing

Synchronous stream ciphers

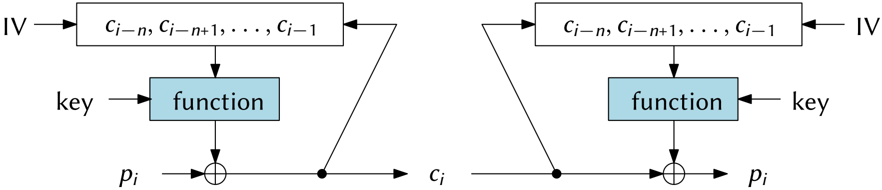


- the most common stream ciphers used in practice
- encryption and decryption are the same
- keystream does not depend on plaintext
- usually binary additive stream ciphers (XOR of plaintext and keystream)

Synchronous stream ciphers 2

- periodic
- require synchronization
  - decryption breaks after losing some bits of ciphertext
- vulnerable to active attacks
  - e.g. changing bits in ciphertext results in change of corresponding plaintext bits
- errors are not propagated
- IV and key must not repeat (otherwise ... two-time pad)
  - be careful of possible keystreams overlaps

Self-synchronizing stream ciphers

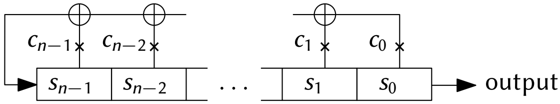


- keystream depends on ciphertext (and therefore on plaintext)
- ability to self-synchronize after the loss of same cipherext
- aperiodic
- hard to analyze, hard to guarantee security properties

Remarks

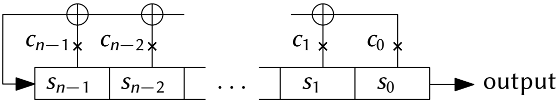
- stream ciphers can be constructed from block ciphers
- specific modes of operation:
  - synchronous: OFB, CTR
  - self-synchronizing: CFB
- Why stream ciphers at all?
  - speed
  - simplicity (HW implementation, constrained environment)
- requirements (preliminary observations):
  - long period
    ...How do you attack stream cipher with short period?
  - good statistical properties
    ...statistical tests of randomness are not sufficient
  - keystream should be *unpredictable* (*indistinguishable* from a random sequence)
    ...KPA $\Rightarrow$ knowing some part of the keystream

Linear Feedback Shift Register (LFSR)



- common primitive for stream cipher construction
- easy to implement in hardware
- the output sequence has good (basic) statistical properties
- easy to analyze

LFSR – notation and function

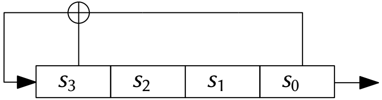


- we focus on binary registers (over GF(2))
- n – length of register
- initial state: $s_{n-1}, \ldots, s_1, s_0 \in \{0, 1\}^n$
- LFSR produces sequence $\{s_i\}_{i \geq 0}$, where for $k \geq 0$:

$$s_{k+n} = c_{n-1}s_{k+n-1} + c_{n-2}s_{k+n-2} + \ldots + c_1 s_{k+1} + c_0 s_k$$

- state after $k \geq 0$ steps: $(s_{k+n-1}, \ldots, s_{k+1}, s_k)$
- characteristic polynomial: $f(x) = x^n + c_{n-1}x^{n-1} + \ldots + c_1 x + c_0$

LFSR – example



$$f(x) = x^4 + x^3 + 1$$

0	0	0	1
1	0	0	0
1	1	0	0
1	1	1	0
1	1	1	1
0	1	1	1
1	0	1	1
0	1	0	1
1	0	1	0
1	1	0	1

0	1	1	0
0	0	1	1
1	0	0	1
0	1	0	0
0	0	1	0
0	0	0	1

LFSR – remarks

- all-zero state never changes $\Rightarrow$ max. period is $2^n - 1$

  LFSR generates a sequence with period $2^n - 1$ *if and only if its characteristic polynomial is a primitive polynomial over* $GF(2)$*.*

- primitive polynomial: irreducible & its root is a generator of multiplicative group in generated finite field
  - irreducible but non-primitive polynomial over GF(2): $x^4 + x^3 + x^2 + x + 1$
- popular polynomials with degree n, where n is a Mersenne prime (i.e. $2^n - 1$ is also prime; e.g. $n = 19, 31, 61, 89, 107, 127$)
  - in this case each irreducible polynomial is primitive
- connection (feedback) polynomial: $1 + c_{n-1}x + \ldots + c_1 x^{n-1} + c_0 x^n$
  - reciprocal polynomial of the characteristic polynomial

LFSR – properties

- some statistical properties (for sequence with maximal period)
  - all states except all-zero state
  - all n-tuples, $(n-1)$-tuples, …almost the same frequency: $\#_0 + 1 = \#_1$, $\#_{00} + 1 = \#_{01} = \#_{10} = \#_{11}, \ldots$
- very easy to predict ($\Rightarrow$ do not use LFSR as a stream cipher)
  - knowledge of $2n$ consecutive bits of output sequence
  - computation of feedback coefficients (system of linear equations)

$$\underline{s_{k+n}} = c_{n-1}\underline{s_{k+n-1}} + c_{n-2}\underline{s_{k+n-2}} + \ldots + c_1\underline{s_{k+1}} + c_0\underline{s_k}$$
$$\underline{s_{k+1+n}} = c_{n-1}\underline{s_{k+n}} + c_{n-2}\underline{s_{k+n-1}} + \ldots + c_1\underline{s_{k+2}} + c_0\underline{s_{k+1}}$$
$$\ldots$$

  - the state is known anyway
  - LFSR can be clocked forward as well as backward

LFSR – synthesis and linear complexity

- input: arbitrary sequence $\{s_i\}_{0 \leq i < L}$
- output: the shortest LFSR generating the input sequence
- trivial solution: solve system of linear equations and enlarge LFSR
- better approach: Berlekamp-Massey algorithm $O(L \cdot n)$
- linear complexity of a sequence:
  - expected for a random sequence: $\sim L/2$
  - large linear complexity is necessary, but not sufficient: $00\ldots01$
- linear complexity profile
  - proper profile necessary, but not sufficient:
    $11010^3 10^7 1\ldots$ with “perfect” profile $\lfloor (L+1)/2 \rfloor$

Increasing linear complexity

- filter generator
  - state or its subset is filtered through a Boolean function
  - properties depend on properties of filter function
- combination generator
  - multiple LFSRs – combination of outputs
  - properties depend on combination function (balanced, high algebraic degree, nonlinearity ...)
  - example: XOR $\Rightarrow$ sum of linear complexities
- linear complexity is not everything ...

Geffe generator – correlation attack

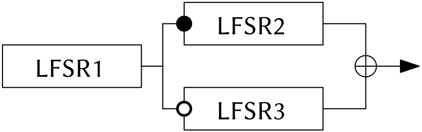
- combination of three LFSRs: $F(x_1, x_2, x_3) = x_1x_2 \oplus (x_1 \oplus 1)x_3$
- brute-force attack – searching through all states: $2^{n_1+n_2+n_3}$
- linear complexity $\leq n_1n_2 + n_1n_3 + n_3$
- $\Pr[x_3 = F(x_1, x_2, x_3)] = \Pr[x_2 = F(x_1, x_2, x_3)] = 3/4$

x_1	x_2	x_3	$F(x_1, x_2, x_3)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

- KPA (i.e. we know the keystream)
- independent attack on LFSR3 state (correct state will correlate with keystream)
- independent attack on LFSR2 state
- finally, attack on LFSR1
- overall $2^{n_3} + 2^{n_2} + 2^{n_1}$

Examples of other constructions based on LSFR

- alternating step generator
  - output of LFSR1 controls if LFSR2 or LFSR3 is clocked (the second one is paused)



- shrinking generator
  - two LFSRs: LSFR1 and LFSR2
  - if output of LSFR1 is 1 then output is the output of LFSR2
  - otherwise there is no output from the generator
- both generators have exponential period and linear complexity

RC4

- Ron Rivest, 1987
- trade secret; posted anonymously to a mailing list in 1994
- internal state $S[0 \dots 255]$ – permutation $\{0, \dots, 255\}$
- key $K[0 \dots k]$ – array of bytes (16 for 128-bit key)
- initialization:

$$
\begin{aligned}
&\text{for } i = 0, \dots, 255\text{: } S[i] = i;\\
&j = 0;\\
&\text{for } i = 0, \dots, 255\text{:}\\
&\quad j = (j + S[i] + K[i \bmod k]) \bmod 256;\\
&\quad \text{swap}(S[i], S[j]);
\end{aligned}
$$

RC4 (2)

- generating keystream:

$$
\begin{aligned}
& i = 0;\ j = 0; \\
& \text{while (is needed):} \\
& \quad i = (i + 1) \bmod 256; \\
& \quad j = (j + S[i]) \bmod 256; \\
& \quad \text{swap}(S[i], S[j]); \\
& \quad \text{output } S[(S[i] + S[j]) \bmod 256];
\end{aligned}
$$

- additive cipher, the output is XOR-ed with plaintext bytes
- first bytes of keystream leak information about key
  - WEP attack (key and IV used as RC4 key)
  - drop some keystrem prefix / different construction of the key

Klein's attack on WEP 1

- WEP (Wired Equivalent Privacy) – security for 802.11 WiFi networks
  - superseded by WPA2 (WiFi Protected Access)
- data frame:

$$\underbrace{\text{IV, padding, ID}_{\text{Rk}},}_{\text{plaintext}}\underbrace{\text{data, ICV}}_{\text{encrypted}}$$

  - IV – initialization vector (3B)
  - ID_{Rk} – Rk's identifier (2 bits)
  - ICV – integrity check value (CRC32)
- RC4 with key $K = \text{IV} \,||\, \text{Rk}$ (Rk – root key)
- Notation:
  - S_i – internal permutation after i-th round ($i \leq 256$ corresponds to initialization)
  - j_i – internal variable j after i-th round
  - X – keystream (obtained by XORing ciphertext and known plaintext data)

Klein's attack on WEP 2

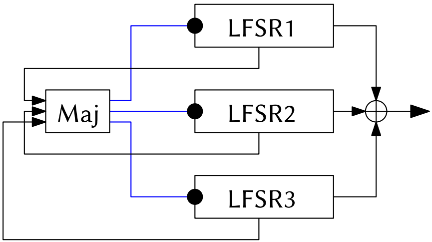
- Klein proved the following property of RC4 ($n = 256$):

$$\Pr[K[i \bmod k] = S_i^{-1}[i - X[i-1]] - (S_i[i] + j_i)] \approx \frac{1.36}{n}$$

  instead of desired $1/n$.
- IV $= K[0], K[1], K[2]$ is known $\Rightarrow$ S_3 and j_3 can be computed
- the value $w = S_3^{-1}[3 - X[2]] - (S_3[3] + j_3)$ is $K[3]$ with probability $\approx \frac{1.36}{n}$
- attacker observes many frames (fixed Rk and different IV) …correct value of $K[3]$ (the first byte of Rk) revealed by statistics
- knowing $K[3] \Rightarrow$ next RC4 round computation: S_4, j_4 …etc.

- improvements for WEP, e.g. PTW attack (2007)
- attack on RC4 in TLS: AlFardan et al. (2013)

A5/1



- used in GSM networks; designed in 1987; reverse engineered in 1999
- LFSRs lengths: 19, 22 a 23 bits (i.e. internal state: 64 bits)
- at least 2 LSFRs clocked in each step (those that agree on selected bits)
- for each frame: first 100 bits are discarded, next 228 bits are used (114 for encryption, 114 for decryption)
- weak cipher, various attacks published (A5/1 replaced by KASUMI in UMTS)

ChaCha20

- high-speed ARX cipher (add-rotate-xor)
- designed by D.J. Bernstein (2008)
- details described e.g. in RFC 8439
- ChaCha20 – specific instance of ChaCha with 20 rounds
- state: 4×4 matrix, elements are 32-bit words
- inputs:
  - key: 256 bits (8 words)
  - nonce (IV): 96 bits (3 words)
  - counter: 32 bits (1 word) $\Rightarrow$ max. 256 GB
- output: 512 bits (64 bytes, 16 words)
- different nonce/counter lengths possible (we follow RFC 8439)

ChaCha20 – initialization and quarter-round

0 const	1 const	2 const	3 const
4 key	5 key	6 key	7 key
8 key	9 key	10 key	11 key
12 cnt	13 nonce	14 nonce	15 nonce

```
QuarterRound(a,b,c,d):
    a += b; d ^= a; d <<<= 16;
    c += d; b ^= c; b <<<= 12;
    a += b; d ^= a; d <<<= 8;
    c += d; b ^= c; b <<<= 7;
```

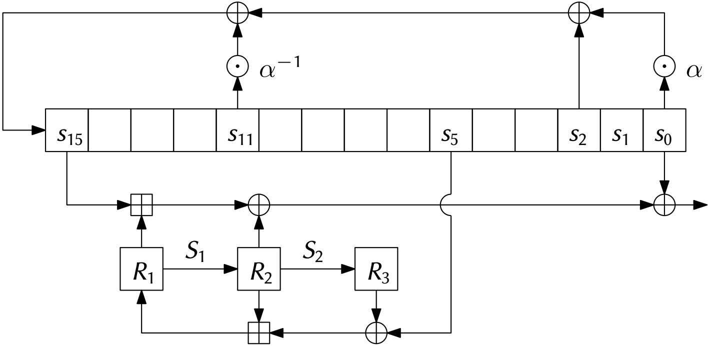
ChaCha20 – block function

- iterate 10 times following two rounds:

```
QuarterRound(0, 4, 8, 12)
QuarterRound(1, 5, 9, 13)
QuarterRound(2, 6, 10, 14)
QuarterRound(3, 7, 11, 15)
QuarterRound(0, 5, 10, 15)
QuarterRound(1, 6, 11, 12)
QuarterRound(2, 7, 8, 13)
QuarterRound(3, 4, 9, 14)
```

- the output state is added (word by word) to the input state $\mapsto$ keystream block
- the output state is used again as an input to the block function

Snow 3G – keystream generator



- SNOW 3G is the base of confidentiality and integrity algorithms UEA2 and UIA2 (for LTE)
- LSFR: 16 32-bit words; S_1, S_2 – s-boxes
- FSM (finite state machine): R_1, R_2, R_3 – 32-bit values
- α is the root of some fixed polynomial